Claim

M_1, M_2, M_3 do not span V, indeed.

$$\mathrm{Span}(M_1,M_2,M_3) = \left\{ a\begin{pmatrix}1&0\\0&0\end{pmatrix} + b\begin{pmatrix}0&1\\0&0\end{pmatrix} + c\begin{pmatrix}0&0\\1&0\end{pmatrix} : a,b \in F \right\}$$

$$= \left\{ \begin{pmatrix}a&b\\c&0\end{pmatrix} : a,b,c \in F \right\} \not\ni \begin{pmatrix}0&0\\0&1\end{pmatrix} \quad \square$$

$\boxed{\text{Exercise: } N_1, N_2, N_3 \text{ also do not span } V.}$

Def^n: A subset $S \subseteq V$ is "linearly dependent" ("bad", "wasteful") if $\exists \alpha_i \in F$ not all of them equal to 0, & $u_i \in S$, s.t. $0 = \sum \alpha_i u_i$,

{distinct u_i's} (otherwise $1 \cdot u_1 - u_1 = 0$)

Otherwise we say S is "linearly independent" ("good")

Example 1 $\left\{ \underbrace{\begin{pmatrix}1\\2\\3\end{pmatrix}}_{u_1}, \underbrace{\begin{pmatrix}4\\5\\6\end{pmatrix}}_{u_2}, \underbrace{\begin{pmatrix}7\\8\\9\end{pmatrix}}_{u_3} \right\} \subseteq \mathbb{R}^3$

S is lin. dep. $2u_2 - u_3 - u_1 = 0$

Example 2 In F^n, $\left\{ u_1 = \begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}, u_2 = \begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}, u_n = \begin{pmatrix}0\\0\\0\\0\\n\end{pmatrix} \right\}$ is linearly independent. Indeed, if $\sum \alpha_i u_i = 0$, then

$$\begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix} = 0_v = \begin{pmatrix}\alpha_1\\\alpha_2\\\vdots\\\alpha_n\end{pmatrix} \Longrightarrow (\alpha_1 = 0) \wedge (\alpha_2 = 0) \wedge \cdots$$

so this isn't a non-trivial comb. so linearly independent $\square$.