$$= \begin{pmatrix} ax_1 + ay_1 \\ \vdots \\ ax_n + ay_n \end{pmatrix} \quad (f5 \text{ n times})$$

Same ∴ Verified VS 7.

$$ax + ay = a\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} + a\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$= \begin{pmatrix} ax_1 \\ \vdots \\ ax_n \end{pmatrix} + \begin{pmatrix} ay_1 \\ \vdots \\ ay_n \end{pmatrix} = \begin{pmatrix} ax_1 + ay_1 \\ \vdots \\ ax_n + ay_n \end{pmatrix}$$

IF $n = 0$ $\quad F^n = \{(\;)\}$ $\quad |F^0| = 1$ (1 element in this V.S.)

$0_{F^0} = (\;)$ ⟶ empty

"the zero vector space"

IF $n = 1$ $\quad F^1 = \{(x_1) : x_1 \in F\} \longleftrightarrow F$

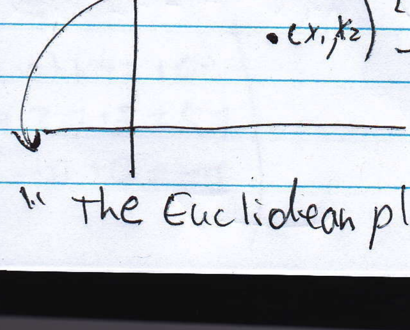
eg $(7) \longleftarrow 7$

$(-2) \longmapsto -2$

∴ F is a v.s. over itself.

in this particular case can multiply vector by vector

$n = 2$, $F = \mathbb{R}$ $\quad V = \mathbb{R}^2 = \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} : x_1, x_2 \in \mathbb{R} \right\}$



"The Euclidean plane"

EXAMPLE 2 $V = M_{m\times n}(F)$

$$= \left\{ \begin{matrix} m\times n \text{ matrices with entries} \\ \text{in } F \end{matrix} \right\}$$

$$= \left\{ A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} : a_{ij} \in F \right\}$$

$$0_V = 0_{M_{m\times n}(F)} = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ \vdots & & & \\ 0 & \cdots & & 0 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ & & \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} + \begin{pmatrix} b_{11} & \cdots & b_{1n} \\ & & \\ b_{m1} & \cdots & b_{mn} \end{pmatrix} = \begin{pmatrix} a_{11}+b_{11} & \cdots & a_{1n}+b_{1n} \\ & & \\ a_{m1}+b_{m1} & \cdots & a_{mn}+b_{mn} \end{pmatrix}$$

$$\begin{pmatrix} A = (a_{ij}), \quad B = (b_{ij}) \\ C = A+B = (c_{ij}) \quad \text{then } c_{ij} = a_{ij} + b_{ij} \end{pmatrix}$$

informal example of scalar multiplication of a matrice

$$7\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 7 & 14 \\ 21 & 28 \end{pmatrix}$$

Claim: These definitions satisfy VS1 – VS8.

(Proof is Tedious)