

Below, you can see the Billiards Puzzle.



The following short script shows a possible way to calculate the corresponding permutation group and its size in Mathematica.

```

a = Cycles[{{1, 2, 3, 4, 5, 6, 7, 8, 9}}];
b = Cycles[{{1, 2, 3, 4, 5, 10}}];
c = Cycles[{{5, 6, 7, 8, 9, 10}}];
group = PermutationGroup[{a, b, c}];
list = GroupElements[group]
Length[list]

```

A very large output was generated. Here is a sample of it:

```

{Cycles[{}], Cycles[{{9, 10}}], Cycles[{{8, 9}}],
Cycles[{{8, 9, 10}}], Cycles[{{8, 10, 9}}], <<3 628 790>>,
Cycles[{{1, 10, 2, 9, 3, 8}, {4, 7}, {5, 6}}],
Cycles[{{1, 10, 3, 8, 2, 9}, {4, 7}, {5, 6}}],
Cycles[{{1, 10}, {2, 9, 3, 8}, {4, 7}, {5, 6}}],
Cycles[{{1, 10, 2, 9}, {3, 8}, {4, 7}, {5, 6}}],
Cycles[{{1, 10}, {2, 9}, {3, 8}, {4, 7}, {5, 6}}]}

```

3 628 800

The configurations are generated by three permutations; moving around the large circle (the cycle denoted by **a**), and moving around the two smaller circles (the cycles **b** and **c**). The program generates the group, lists its elements and gives us that the corresponding subgroup of $\text{Sym}(10)$ has 3 628 800 elements.