<u>Differential Equation</u>: Equation in which the unknown is a function and we are given some relation between that function and its derivatives, all evaluated at the same point.

<u>Example</u>

1. $y' = y \iff \forall x,\ f'(x) = f(x)$

   $y = e^x \to y = ce^x$

   Initial condition : $y(0) = \frac{22}{7}$

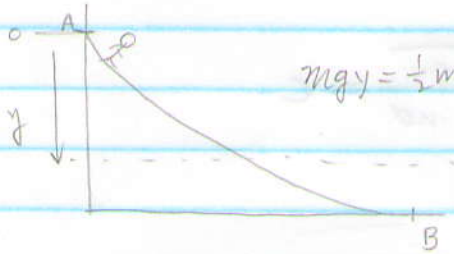
   $\rightsquigarrow$ fixes the solution to be $y(x) = \frac{22}{7}e^x$

2. $y' = y + e^x$

   $y = xe^x \to$ found a special solution. Are there more?

3. $3 + \dfrac{y e^{(y-y')^2}}{\cos(x+y')} = y''$

<u>The Brachistochrone Problem</u>



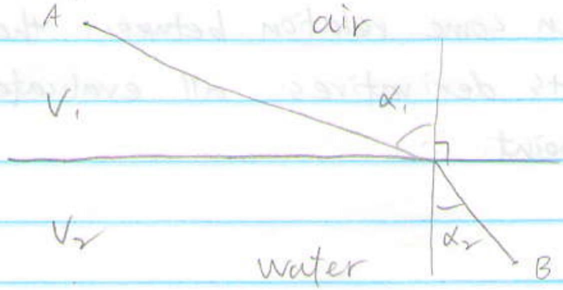
$mgy = \frac{1}{2}mv^2$

$v = \sqrt{2gy}$

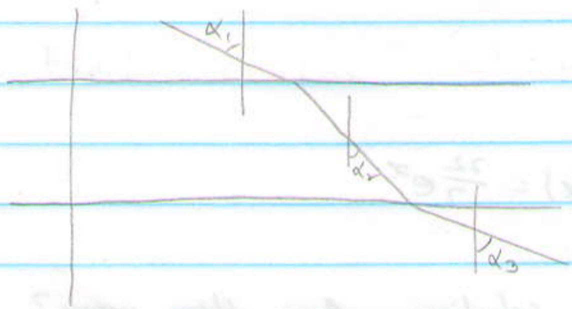
<u>Fermat's Principle</u>

When light travels from A to B, it picks the quickest route.

example (Snell's law)



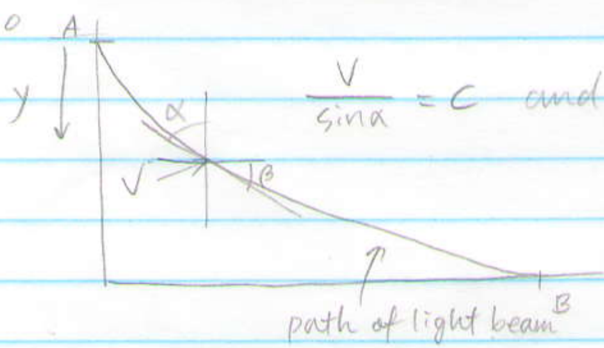
$$\frac{\sin\alpha_1}{\sin\alpha_2} = \frac{V_1}{V_2} \iff \frac{V_1}{\sin\alpha_1} = \frac{V_2}{\sin\alpha_2}$$



$$\frac{V_1}{\sin\alpha_1} = \frac{V_2}{\sin\alpha_2} = \frac{V_3}{\sin\alpha_3}$$

$\frac{V}{\sin\alpha}$ is constant.

The Brachistochrone Problem



$$\frac{V}{\sin\alpha} = c \text{ and } V = \sqrt{2gy} \Rightarrow \frac{\sqrt{2gy}}{\sin\alpha} = c$$

$$y' = \tan\beta$$

$$\frac{1}{\cos^2\beta} = \frac{\sin^2\beta + \cos^2\beta}{\cos^2\beta} = \tan^2\beta + 1 \Rightarrow \cos^2\beta = \frac{1}{1+\tan^2\beta}$$

$$\sin\alpha = \sin(90-\alpha) = \cos\beta = \frac{1}{\sqrt{1+\tan^2\beta}} = \frac{1}{\sqrt{1+y'^2}}$$

$$\Rightarrow \text{Need to solve } \frac{\sqrt{2gy}}{1/\sqrt{1+y'^2}} = c \iff \boxed{y\cdot(1+y'^2) = d}$$