(or)

P.12 $a \times b = 0$ iff $(a=0) \vee (b=0)$
$\Longleftrightarrow$

Proof:

$\Longleftarrow$ if $a=0$ then $a \times b = 0 \times b \overset{P8}{=} 0$ (skipping $\overset{F_1}{\text{commutative}}$ law ie $a \times b = b \times 0 = 0$)

if $b=0$ then $a \times b = a \times 0 \overset{P8}{=} 0$ $\blacksquare$

$\Longrightarrow$ Assume $a \times b = 0$ if $a=0$ ✓.
otherwise, $a \neq 0$ and we have $a \times b = 0 \overset{P8}{=} a \times 0$
by multiplication cancellation (P_2) $\cancel{a} \times b = \cancel{a} \times 0$ $\therefore b=0$. $\square$

There are infinite properties one could prove ie $b \neq 0, c \neq 0$ $\dfrac{\frac{a}{c}}{\frac{b}{c}} = \dfrac{a}{b}$ etc

eg $(a+b)(a-b) = a^2 - b^2$

Proof

$a(a-b) + b(a-b) = a(a+(-b)) + b(a+(-b))$
$= a \times a + a \times (-b) + b \times a + b \times (-b)$ etc.

$-2, -1, 0, 1, 2 := 1+1$ (2 may or may not be 0)
$3 := 2+1$, $4 := 3+1$.

ie Field w/ 2 elements, you can say it doesn't have 2 or there is 2 but $0=2$, in that sense every field contains all integers even if they repeat themselves ie $0, 1, \underset{2}{0}, \underset{3}{1}, \underset{4}{0}, \underset{5}{1}$ etc

Theorem means "unique"

"iota, function"

$\exists!\ \iota\ \mathbb{Z} \longrightarrow F$ s.t.

1. $\iota(0_\mathbb{Z}) = 0_F$, $\iota(1_\mathbb{Z}) = 1_F$

2. $\forall m,n \in \mathbb{Z}$ $\quad \iota(m +_\mathbb{Z} n) = \iota(m) +_F \iota(n)$

3. $\forall m,n \in \mathbb{Z}$ $\quad \iota(m \times_\mathbb{Z} n) = \iota(m) \times \iota(n)$

Set $\iota(0) = 0_F$, $\iota(1) = 1_F$ — because had to satisfy prop 2... no choice how to set ι ... in that sense unique.

$\iota(2_\mathbb{Z}) = \iota(1+1) = \iota(1) + \iota(1) = 1_F + 1_F$

$\iota(3) = \iota(2+1) = \iota(2) + \iota(1) = \iota(2) + 1$

[**Excersize:** Figure out how to define "ι" on negative integers]

Claim: the ι just defined satisfies properties 1–3.

(won't prove because requires induction etc).

Ex $\iota_{F_2} \cdot 27 \longrightarrow \iota(26) = \iota(26+1) = \iota(26) + 1$

$= \iota(2 \times 13) + 1$

$= \iota(2) \times \iota(13) + 1$ (by 2)

$= (1+1) \times \iota(13) + 1$

$= 0 \times \iota(13) + 1$

$= 0 + 1$

$= 1$