Chapter 16 Polynomial Rings

Definition Ring of Polynomials over R

Let R = commutative ring

Then $R[x] = \{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 : a_i \in R, n \in Z+\}$ is called the *ring of polynomials over* R in the *indeterminate* x

Say $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$, where $a_n \neq 0$

$\Rightarrow$ *degree of* f(x) = n

Leading coefficient of f(x) = a_n

If a_n = unity of R = multiplicative identity of R

Then f(x) is a *monic polynomial*

Say f(x) = 0 $\Rightarrow$ f(x) has *no degree*

Say f(x) = constant, then f(x) are called *constant*

Thm 16.1 D an Integral Domain Implies D[x] an Integral Domain

Thm 16.2 Division Alogorithm for F[x]

Let:

1. F = field
2. f(x), g(x) $\neq 0 \in$ F[x]

Then: $\exists$ unique polynomials q(x), r(x) $\in$ F[x] *such that* f(x) = g(x)q(x) + r(x) *in which* r(x) = 0 *or* deg r(x) < deg g(x)

Let:

1. **F = field**
2. **a $\in$ F, f(x) $\in$ F[x]**

Then 1: Corollary 1 The Remainder Theorem

f(a) = remainder when f(x) is divided by (x – a)

Then 2: Corollary 2 The Factor Theorem

a = zero of f(x) *if and only if* (x – a) is a factor of f(x)

Corollary 3 Polynomials of Degree *n* have at most *n* Zeros

Definition A *principal ideal domain* (PID) *is an* integral domain R in which every ideal has the form $\langle a \rangle = \{ra : r \in R\}$

Thm 16.3 F = field, *Then* F[x] = PID

Thm 16.4 Criterion for I = $\langle g(x) \rangle$

Let F = field, I = non-zero ideal in F[x], g(x) $\in$ F[x]

Then I = $\langle g(x) \rangle$ *if and only if* g(x) = polynomial of minimum degree in I