Do not turn this page until instructed.

Math 1300 Geometry and Topology

Term Exam 2

University of Toronto, February 11, 2007

Solve the 4 problems on the other side of this page.
Each problem is worth 30 points.
You have an hour and fifty minutes to write this test.

Notes.

- No outside material other than stationary is allowed.
- **Neatness counts! Language counts!** The *ideal* written solution to a problem looks like a page from a textbook; neat and clean and made of complete and grammatical sentences. Definitely phrases like "there exists" or "for every" cannot be skipped. Lectures are mostly made of spoken words, and so the blackboard part of proofs given during lectures often omits or shortens key phrases. The ideal written solution to a problem does not do that.
- Advance apology: It may take me a while to grade this exam; sorry.

Good Luck!

Solve the following 4 problems. Each problem is worth 30 points. You have an hour and fifty minutes. **Neatness counts! Language counts!**

Problem 1 "Compute". Let $\omega \in \Omega^2(\mathbb{R}^3_{xyz})$ be $\omega = ydx \wedge dz$, and orient $\mathbb{R}^3_{xyz}$ using the order (x, y, z). Let R be the rectangle $[-\frac{\pi}{2}, \frac{\pi}{2}]_\theta \times [0, 2\pi]_\phi$ oriented using the order (θ, ϕ). Let $\lambda : R \to \mathbb{R}^3_{xyz}$ be given by $\lambda(\theta, \phi) = (\cos\theta\cos\phi, \sin\theta, \cos\theta\sin\phi)$.

1. Compute $\lambda^\star\omega$.
2. Compute $\displaystyle\int_R \lambda^\star\omega$.
3. Compute $d\omega$.
4. Compute $\displaystyle\int_{[x^2+y^2+z^2\leq 1]} d\omega$.

Problem 2 "Reproduce". State precisely and prove in detail the theorem about existence and uniqueness of lifts of maps $f : Y \to B$, where B is the basis of a covering $p : X \to B$.

Problem 3 "Think". Let Y be the space obtained from a triangle by identifying its three edges, where all three edges are oriented counterclockwise. (Alternatively, $Y = \{z \in \mathbb{C} \colon |z| \leq 1\}/(z \sim e^{2\pi i/3}z$ whenever $|z| = 1))$.

1. Compute $\pi_1(Y)$, quoting the theorems you use along the way.
2. Prove that every map $Y \to \mathbb{R}P^2$ lifts to a map $Y \to S^2$, or find one that doesn't.

Problem 4 "Sketch". Sketch the derivation of the four Maxwell equations, along with the necessary condition on the charge-current, using differential forms and starting from the least action principle.

Good Luck!