Properties of det:

② det is linear in all rows.
(i.e., is multilinear in the rows)

③ if $A$ has first two rows equal, then $\det A = 0$

④ if $A$ has any two rows equal, then $\det A = 0$

⑤ if $A'$ is gotten from $A$ by switching the first two rows, then $\det(A') = -\det(A)$

Pf: $\det(A') = \sum_{j=1}^{n} (-1)^{1+j} A'_{1j} \det(A'_{1j})$ ⊖

Since $A'_1 = A_2$, and $A'_2 = A_1$,
it follows $A'_{1j} = A_{2j}$, and $A'_{1j} = A_{2j}$

$A'_{1j} = \begin{bmatrix} - A_1 - \\ - A_3 - \\ \vdots \\ - A_n - \end{bmatrix} = \begin{bmatrix} - A_1 - \\ - A_3 - \\ \vdots \\ - A_n - \end{bmatrix}$

$$\ominus \sum_{j=1}^{n} (-1)^{1+j} A_{2j} \det(A_{\hat{2}\hat{j}})$$

<u>note</u>: looks almost like the defn of det expanded along $2^{nd}$ row, except the power is wrong, should be $(-1)^{2+j}$.

$$= (-1)^{1} \sum_{j=1}^{n} (-1)^{2+j} A_{2j} \det(A_{\hat{2}\hat{j}})$$

$$= -\det(A)$$

⑥ If $A'$ is gotten from $A$ by switching any two rows, then $\det(A') = -\det(A)$.

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ \vdots \end{bmatrix}$$

Properties of det!

Sps rows $A_r, A_s$ are switched, $r \neq s$.

Pf: $n=2$, then it's clear:

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, then $A' = \begin{bmatrix} a_{21} & a_{22} \\ a_{11} & a_{12} \end{bmatrix}$

$\det A = a_{11}a_{22} - a_{21}a_{12}$, and $\det A' = a_{21}a_{12} - a_{11}a_{22}$

$= -(a_{11}a_{22} - a_{21}a_{12})$

$= -\det(A)$

If $n \geq 3$, then let $A_i$ be a row s.t. $i \neq r, s$.

Then expand along $A_i$.

$$\det(A') = \sum_{j=1}^{n} (-1)^{i+j} A'_{ij} \det(A'_{\hat{i}\hat{j}})$$

Since $A_i$ wasn't switched, $A'_i = A_i$.

and $A'_{\hat{i}\hat{j}} \in Mat_{(n-1)\times(n-1)}(\mathbb{F})$.

By applying induction on $n$, we have

$$\det\left(A'_{\hat{i}\hat{j}}\right) = -\det\left(A_{\hat{i}\hat{j}}\right),$$

and so

$$\det(A') = \sum_{j=1}^{n} (-1)^{i+j} A_{ij} (-1) \det\left(A_{\hat{i}\hat{j}}\right)$$

$$= -\det(A)$$