

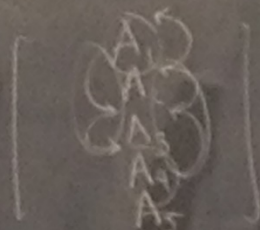
$$\textcircled{5} \sum_{j=1}^n (-1)^{1+j} A_{1j} \det(A_{2j})$$

note: looks almost like the defn of det expanded along 2nd row, except the power is wrong, should be $(-1)^{2+j}$.

$$= (-1)^{1+1} \sum_{j=1}^n (-1)^{2+j} A_{1j} \det(A_{2j})$$

$$= -\det(A)$$

⑥ If A' is gotten from A by switching any two rows, then $\det(A') = -\det(A)$.



Properties of \det

Sp. rows A_r, A_s are switched, $r \neq s$.

If $n=2$, then it's clear:

$$\text{if } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \text{ then } A' = \begin{bmatrix} a_{21} & a_{22} \\ a_{11} & a_{12} \end{bmatrix}$$

$$\det A = a_{11}a_{22} - a_{21}a_{12}, \text{ and } \det A' = a_{11}a_{22} - a_{21}a_{12} \\ = -(a_{11}a_{22} - a_{21}a_{12}) \\ = -\det(A)$$

If $n \geq 3$,
Then expand

$$\det(A')$$

Since A_i
and A_j

If $n \geq 3$, then let A_i be a row with $i \neq r, s$.

Then expand along A_i .

$$\det(A') = \sum_{j=1}^n (-1)^{i+j} A'_{ij} \det(A'_{ij})$$

Since A_i wasn't switched, $A_i = A_i$.

and $A_{ij} \in \text{Mat}_{(n-1) \times (n-1)}(\mathbb{F})$.

By applying induction on n , we have

$$\det(A_{ij}^{\circ}) = -\det(A_{ij}),$$

and so

$$\begin{aligned} \det(A') &= \sum_{j=1}^n (-1)^{i+j} A_{ij} \det(A_{ij}^{\circ}) \\ &= -\det(A) \end{aligned}$$