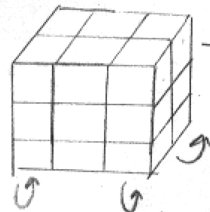
Part 1

I chose the 2x3x3 rubik's cube.

3 rotations of 90° σ_5

G



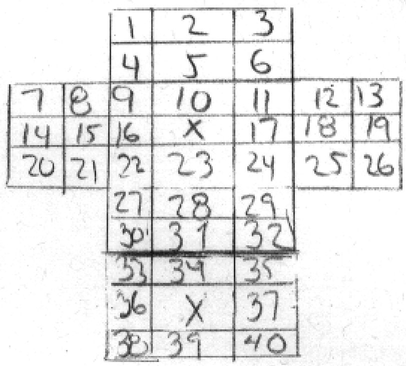
1 rotation of 180° σ_1

1 rotation of 180° σ_2

1 rotation of 180° σ_3

1 rotation of 180° σ_4

Thus, there are 5 generators of the group G.
Let's label each side



The generators are

$\sigma_1 = (9\ 40)(10\ 39)(11\ 38)(12\ \ 7)(13\ \ 8)(6\ 1)(4\ 3)(5$

$\sigma_2 = (22\ 35)\ (23\ 34)(24\ 33)(25\ 20)(26\ 21)(29\ 30)(27\ 32)($

$\sigma_3 = (11\ 35)(17\ 37)\ (24\ 40)\ (6\ 32)\ (3\ 29)(12\ 26)(18\ 19)(25$

$\sigma_4 = (9\ 33)\ (16\ 36)\ (22\ 38)(\ 4\ \ 30)\ (1\ 27)\ (7\ 21)\ \ (8\ 20)\ (14$

$\sigma_5 = (11\ \ 9\ 22\ 24)\ (10\ 16\ 23\ 17)\ (25\ 6\ 8\ 27)\ \ (18\ 5\ 15\ 28$

$(12\ \ 4\ 21\ 29)$

So we are now ready to count the number of configurations.

I did a first code, where I decided to def the multiplication and just multiply all the elements together (see code 1). But this Code is too long to run so I decided to use Prof Bar natan's code (see code 2) and obtained as a result that $|G| = 1\,625\,702\,400$.

Remark: $1\,625\,702\,400 = 8!\,8!$

This result makes sence since there are 8 corner pieces and 8 edge pieces. Each of the pieces can go every where, but it is impossible to change the "orientation". For example the cube 10-5 cannot become the cube 5-10.* This is due to the fact that the 3x3x2 Rubik's Cube is asymmetric.

So $|G| = 8!\,8!$.

*which mean the cube at the same place but 5 on the fron face and 10 on the top face

First Code

```
sigma1 = {6, 5, 4, 3, 2, 1, 12, 13, 40, 39, 38, 7, 8, 14, 15, 16, 17, 18, 19,
   20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 11, 10, 9}

sigma2 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19,
   25, 26, 35, 34, 33, 20, 21, 32, 31, 30, 29, 28, 27, 24, 23, 22, 36, 37, 38, 39, 40}

sigma3 = {1, 2, 29, 4, 5, 32, 7, 8, 9, 10, 35, 26, 25, 14, 15, 16, 37, 19, 18,
   20, 21, 22, 23, 40, 13, 12, 27, 28, 3, 30, 31, 6, 33, 34, 11, 36, 17, 38, 39, 24}

sigma4 = {27, 2, 3, 30, 5, 6, 21, 20, 33, 10, 11, 12, 13, 15, 14, 36, 17, 18,
   19, 8, 7, 38, 23, 24, 25, 26, 1, 28, 29, 4, 31, 32, 9, 34, 35, 16, 37, 22, 39, 40}

sigma5 = {1, 2, 3, 21, 15, 8, 7, 27, 22, 16, 9, 4, 13, 14, 28, 23, 10, 5, 19,
   20, 29, 24, 17, 11, 6, 26, 25, 18, 12, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40}

mult1[p1_List, p2_List] := Table[p2[[p1[[m]]]], {m, Length[p1]}]

group = {sigma1, sigma2, sigma3, sigma4, sigma5}

jump = 1; For[i = 1, jump ≠ 0, i = i + jump, group = Union[group]; k = Length[group];
 For[j = 1, j ≤ k, j++, AppendTo[group, mult1[group[[i]], group[[j]]]];
  AppendTo[group, mult1[group[[j]], group[[i]]]]]; jump = Length[Union[group]] - k]

Length[group]
```

Second Code

```
gs = {perm1 = P[6, 5, 4, 3, 2, 1, 12, 13, 40, 39, 38, 7, 8, 14, 15, 16, 17, 18, 19, 20,
      21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 11, 10, 9],
    perm2 = P[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 25, 26,
      35, 34, 33, 20, 21, 32, 31, 30, 29, 28, 27, 24, 23, 22, 36, 37, 38, 39, 40],
    perm3  = P[1, 2, 29, 4, 5, 32, 7, 8, 9, 10, 35, 26, 25, 14, 15, 16, 37, 19, 18, 20,
      21, 22, 23, 40, 13, 12, 27, 28, 3, 30, 31, 6, 33, 34, 11, 36, 17, 38, 39, 24],
    perm4 = P[27, 2, 3, 30, 5, 6, 21, 20, 33, 10, 11, 12, 13, 15, 14, 36, 17, 18, 19,
      8, 7, 38, 23, 24, 25, 26, 1, 28, 29, 4, 31, 32, 9, 34, 35, 16, 37, 22, 39, 40],
    perm5 = P[1, 2, 3, 21, 15, 8, 7, 27, 22, 16, 9, 4, 13, 14, 28, 23, 10, 5, 19, 20,
      29, 24, 17, 11, 6, 26, 25, 18, 12, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40]};

(RecursionLimit = 2^16; n = 40; P /: p_P ** P[a__] := p[[{a}]];
  Inv[p_P] := P @@ Ordering[p]; Feed[P @@ Range[n]] := Null;
  Feed[p_P] := Module[{i, j}, For[i = 1, p[[i]] == i, ++i]; j = p[[i]];
    If[Head[s[i, j]] === P, Feed[Inv[s[i, j]] ** p], (*Else*) s[i, j] = p; Do[If[
       Head[s[k, l]] == P, Feed[s[i, j] ** s[k, l]]; Feed[s[k, l] ** s[i, j]]], {k, n}, {l, n}]]]);

(Feed[#]; Product[1 + Length[Select[Range[n], Head[s[i, #]] === P &]], {i, n}]) & /@ gs

{2, 4, 96, 192, 1625702400}
```