Nov 2, 2006.

MAT 240

$$\mathcal{L}(V,W) \longrightarrow M_{m\times n}(F)$$
dim = n, m
basis = β, γ
$$T \longmapsto [T]_\beta^\gamma = A$$

$T \to A$
$S \mapsto B$
$T+S \to A+B$
$7T \to 7A$

$$A = \begin{pmatrix} a_{11} & & a_{1n} \\ [Tv_1]_\gamma & \cdots & [Tv_n]_\gamma \\ a_{m1} & & a_{mn} \end{pmatrix} \iff \forall_j \quad Tv_j = \sum_{k=1}^{m} a_{kj} w_k$$

fundamentals of last class

$$\begin{tikzcd} U \arrow[r, "S"', "B"'] \arrow[rr, bend right, "T\circ S \to ? \Rightarrow C"'] & V \arrow[r, "T", "A"'] & W \end{tikzcd}$$
$(u_i)_{i=1}^{\ell}$, $(v_j)_{j=1}^{n}$, $(w_k)_{k=1}^{m}$

$$M_{m\times n} \Rightarrow A = (a_{kj}) \longrightarrow Tv_j = \sum_{k=1}^{m} a_{kj} w_k$$

$$M_{n\times \ell} \Rightarrow B = (b_{ji}) \longrightarrow Su_i = \sum_{j=1}^{n} b_{ji} v_j$$

$$M_{m\times \ell} \Rightarrow C = (c_{ki}) \to (T\circ S)(u_i) = \sum_{k=1}^{m} \underbrace{c_{ki}}_{?} w_k$$

$$\begin{aligned}(T\circ S)(u_i) &= T(S(u_i)) = T\left(\sum_{j=1}^{n} b_{ji} v_j\right) \\ &= \sum_{j=1}^{n} b_{ji}\, T(v_j) = \sum_{j=1}^{n} b_{ji} \sum_{k=1}^{m} a_{kj} w_k \\ &= \sum_{k=1}^{m} \underbrace{\left(\sum_{j=1}^{n} a_{kj} b_{ji}\right)}_{c_{ki}} w_k\end{aligned}$$

$$c_{ki} = \sum_{j=1}^{n} a_{kj} b_{ji}$$

<u>Def</u>: In this case, we say that C is the "product" of A & B, $C = A\cdot B$ (matrix of $\underset{A}{T}\circ \underset{B}{S}$) defined for matrices A & B for which (# of columns of A) = (# of rows of B)

<u>Examples</u> $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ (3 cols) $\qquad B = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 1 & 1 & -1 \end{pmatrix}$

$A \in M_{m\times n}$, $B \in M_{n\times \ell} \Rightarrow AB \in M_{m\times \ell}$

AB has as many rows as $A \to 2$
as many cols as $B \to 3$

$$2\left\{\underbrace{\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & c_{23} \end{pmatrix}}_{3}\right. \qquad C_{23} = \sum_{j=1}^{3} a_{2j} b_{j3}$$

$$= a_{21}\cdot b_{13} + a_{22}\cdot b_{23} + a_{23}\cdot b_{33} = 4\cdot 2 + 5\cdot 0 + 6(-1) = 2$$

a better way of multiplying matrices

$A \cdot B$

$$\begin{array}{cc} & (B) \\ (A) & \boxed{} \end{array} \qquad \begin{array}{cc} & \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 1 & 1 & -1 \end{pmatrix} \\ \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} & \begin{pmatrix} 4 & 1 & -1 \\ 10 & 1 & 2 \end{pmatrix} \end{array}$$

or $(2 \cdot 1) + (2 \cdot 0) + (3 \cdot -1) = -1$

matrices $\to 0, 1, +, \cdot$

$\to$ Are matrices a field?

NO, though almost.

- Why not?

1) $\times$ is not always defined $\to$ then restrict to 7×7 {

2) many matrices other than 0 have no inverse

3) $AB \neq BA$

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \overset{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}}{\begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}} = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$$ rows were swapped

$$\#$$

$$B \cdot A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \overset{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}{\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$$ columns were swapped

$$(2) \overset{(3)}{(6)} \qquad M_{1 \times 1} = F$$