Mar. 30th.

$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \sim \omega = \sum a_i dx_i \qquad \gamma = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix}$.

$$\int_\Gamma \omega = \int_{[0,1]} \gamma^* \omega = \int_{[0,1]} \sum a_i d\gamma_i = \int_{[0,1]} \sum a_i \dot{\gamma}_i dt$$

$$= \int a \cdot \dot{\gamma} = \int_{[0,1]} a \cdot \vec{T} \|\dot{\gamma}\| = \int a \cdot \vec{T} dv$$

Curve Γ function f. $\quad \int_\Gamma (\text{grad} f) \cdot \vec{T} dv = f(\gamma(1)) - f(\gamma(0))$

$$\int_S \omega = \int_{D^2_{x,y}} \sigma^* \omega$$

$\omega = b_1 dx_{23} + \text{c.p.}$

$$\sigma^* = b_1 d\sigma_2 \wedge d\sigma_3 \overset{+\text{c.p.}}{\curlyvee} = b_1 \left(\frac{\partial \sigma_2}{\partial x} dx + \frac{\partial \sigma_2}{\partial y} dy\right) \wedge \left(\frac{\partial \sigma_3}{\partial x} dx + \frac{\partial \sigma_3}{\partial y} dy\right) + \text{c.p.}$$

$$= b_1 \left(\frac{\partial \sigma_2}{\partial x} \cdot \frac{\partial \sigma_3}{\partial y} - \frac{\partial \sigma_3}{\partial x} \cdot \frac{\partial \sigma_2}{\partial y}\right) dx \wedge dy + \text{c.p.}$$

$$= b_1 \left(\frac{\partial \sigma}{\partial x} \times \frac{\partial \sigma}{\partial y}\right)_1 + \text{c.p.} = b \cdot \left(\frac{\partial \sigma}{\partial x} \times \frac{\partial \sigma}{\partial y}\right) dx \wedge dy$$

$$\int \omega = \int_{D^2} \sigma^* \omega = \int b \cdot \vec{n} \cdot V\left(\frac{\partial \sigma}{\partial x}, \frac{\partial \sigma}{\partial y}\right) = \int_S b \cdot \cancel{\vec{n}} \, dv$$

$V_1 \times V_2 = V(V_1, V_2)\vec{n}$

$\boxed{1}$ $k = 1$

$$\int (\text{grad} f) \cdot \vec{T} dv = \oint f(\gamma(1)) - f(\gamma(0)).$$

$k = 2$.

$$\int_S (\text{curl}\, a) \cdot n \, dv = \int_S d\omega = \int_{\partial S} \omega = \int_{\partial S} a \cdot \vec{T} dv$$

S local spanning in the plane ∂S of S.

circulation of a along ∂S.

$S^{n-1} \xhookrightarrow{i} \mathbb{R}^n$ w/ standard orientation.

$$\omega = i^* \left(\sum_i (-1)^{i-1} x_i \, \widehat{dx_i}\right) \in \Omega^{top}(S^{n-1}).$$

Prove that if $(v_1, \ldots, v_{n-1})$ positively oriented basis of $T_x S^{n-1}$

Then. $\omega(v_1 \ldots v_{n-1}) = V$.

$$\int_{S^{n-1}} f\omega = \int_{S^{n-1}} f \, dV$$

$$\left(\sum (-1)^{i-1} x_i \, dx_1 \wedge \ldots \widehat{dx_i} \ldots \wedge dx_n\right) ((x; v)_1, \ldots (v_{n-1}))$$

$$= \sum (-1)^{i-1} x_i \det \begin{pmatrix} v_{11} & v_{1\,n-1} \\ \hat{v}_i & \hat{v}_{i\,n-1} \end{pmatrix}$$

$$= \det(x, v_1, \ldots, v_{n-1}) = \sqrt{\det(x, v_1, \ldots v_{n-1})^2}$$

$$= \sqrt{\det\begin{pmatrix} x^T \\ v_1^T \\ \vdots \\ v_n^T \end{pmatrix} \det(x\, v_1 \ldots v_n)} = \sqrt{\det\begin{pmatrix} \langle x, x\rangle & \langle x, v_1\rangle & \ldots \\ & \langle v_1, v_1\rangle & \end{pmatrix}}$$

$$\langle x, x\rangle = 1$$

$$\Rightarrow \sqrt{\det(v_1 \ldots v_{n-1})^T \det(v_1 \ldots v_{n-1})}$$

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