Problem Set 1. Due Wednesday January 25

1. (a) Prove that the remainder of a cube of any integer modulo 9 can only be 0, 1, or 8.
(b) Without using a calculator, show that the number 123456 is not a cube of any integer.

2. (a) Find a pair of integers $u, v \in \mathbb{Z}$ such that $56u + 135v = 1$.
(b) Does there exist a pair of integers $u_1, v_1 \in \mathbb{Z}$ such that $56u_1 + 135v_1 = 10$?
(c) Can you find *all* the pairs of integers (u, v) satisfying the condition in part (a)?

3. Prove that for any nonzero integers a, b, $(a + b)^2 \equiv (a - b)^2 \mod a$.

4. Let a, b be integers, and $d = (a, b)$.
Prove that $d \mid a^3b + a^2b^2 + ab^3$.
Is it true that $d^4 \mid a^3b + a^2b^2 + ab^3$?
Is it true that $d^5 \mid a^3b + a^2b^2 + ab^3$?

5. Find the remainder of $3^{27} + 15$ modulo 7.

6. Solve the equation $34x = 1$ in $\mathbb{Z}_{97}$.

7. Solve the system of congruences

$$\begin{aligned} x &\equiv 1 \mod 3 \\ x &\equiv 2 \mod 5 \\ x &\equiv 3 \mod 7. \end{aligned}$$