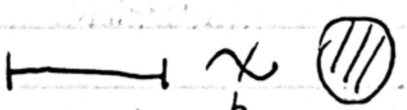
Jan. 25 Wed Hour 45. Manifolds / Boundaries

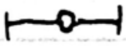
~~R~~ Read along : Sec 23. 24

Q: Is  a ball homeomorphism to  ? (No)
Precisely, is $B^k = \{x \in \mathbb{R}^k : \|x\| < 1\}$ ~~to~~ homeo. $HB^k = \{x \in B^k : x_k \geq 0\}$?

Aside: Is  a ball homeomorphism to  (a sphere)? (No)
Is B^k homeo. to B^n for $n \neq k$?

$k=1, n>1$: 

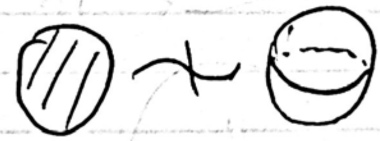
pf: remove a point in $\mathbb{R}^k = \mathbb{R}$, the interval becomes disconnected.



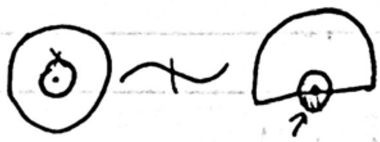
$\partial M (k-1)$ manifold.

is $(-1,1) \sim [0,1)$? No, remove 0 in $[0,1)$ doesn't break.

Sln: In ~~LHS~~ RHS, $\exists$ a point ~~whose~~ (whose) removal keeps continuity, but not in LHS.

$k=2, n>2$: 

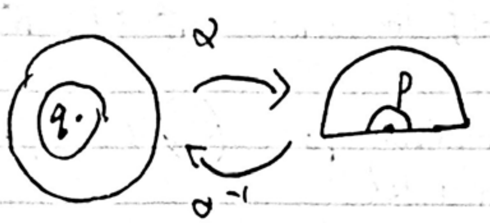
Sln: Simple connectivity (near the end of MAT327 TT2.

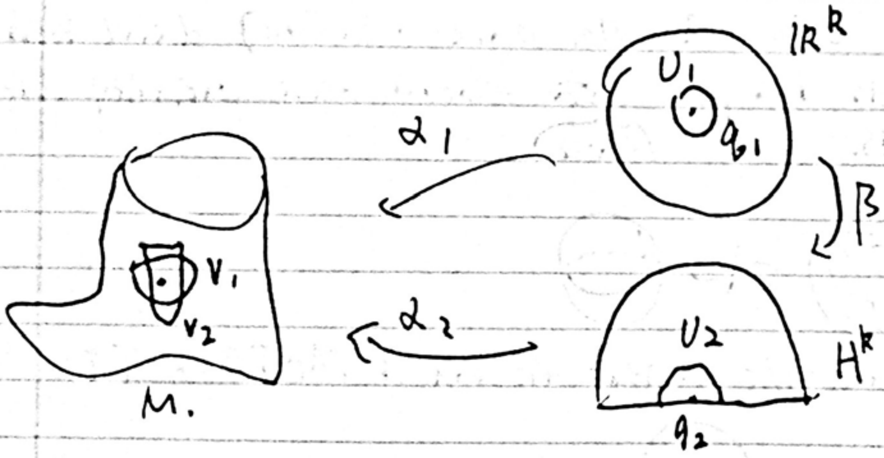
$k=2$:  MAT327

$k=3$, $n>k$  MAT1301, near the end - hard

$k \geq 3$. hard.

Now homeo → diffumo.. means $\exists\, a : X \to Y$ diffable.
X diffable to Y, with diffable inverse. (much easier)

$B^k \sim HB^k$	$B^k \sim B^n \cdot k \neq n$
 α^{-1} works in small nbd. pick a vector in α, $\exists \to$ lin. trans so, no decrease (rank) ↑ invertible	If $\alpha : B^k \to B^n$ $(k \neq n)$ diffable with α^{-1} diffable, then $(d\alpha)^{-1} = d(\alpha^{-1})$. in partical $(d\alpha)^{-1}$ exists but no $n \times k$ matrix is invertible.



transition function

$v_1 \cap v_2$ is an open nbd of P in M.
Let $(i = 1.2)$, $U_i' = \alpha_i^{-1}(V_1 \cap V_2)$

Then U_i' are open nbds of q_i in $\mathbb{R}^k / H^k$.

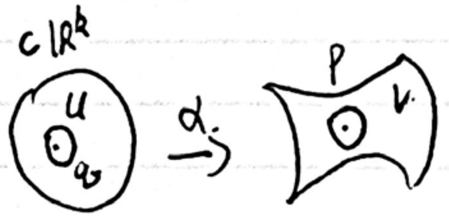
Let $\beta = \alpha_2^{-1} \circ \alpha_1 : U_1' \to U_2'$ is 1-1. onto (we know), is a homeo.

By Boi, get a contradiction.

If we know that ~~$\exists$~~ α_2^{-1} is diffable, we'd be done without 130l. IS α_2^{-1} diffable?

Proposition: If ~~(M~~ M^k is a closed. C^r manifold in $\mathbb{R}^n$.
and $\alpha: U \subseteq \mathbb{R}^k \to V \subseteq M^k$ is a coordinate patch, meaning
have ~~tt~~ $U \to V$, ~~diffable~~, differential, $dF \cdot \alpha$ has max rank.

Then $\alpha^{-1}: V \to U$ is also C^r
$V \subseteq \mathbb{R}^n$

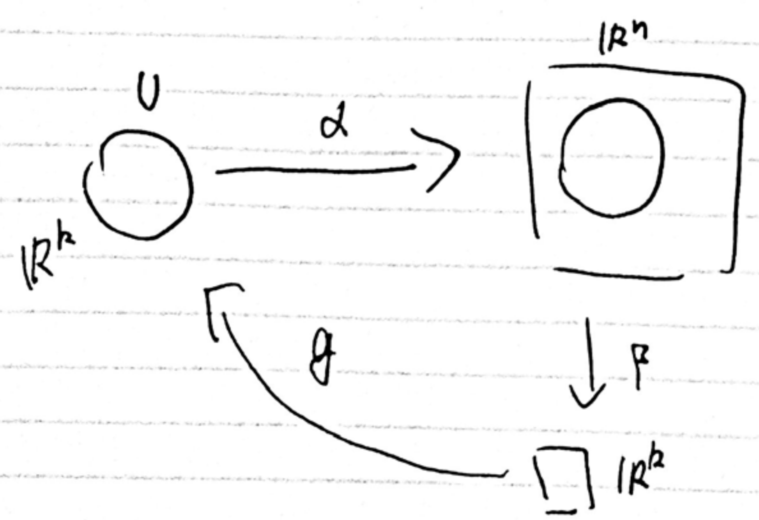


($\forall$ p in the patch, be able to extend to α^{-1}, a n-dim ball about the point)

$D\alpha(q_0)$ has rank K.

for convience, assume the first k rows of $D\alpha(q_0)$ are independent

next, project to first k rows, then $Im(U)$ is a nice set
2 sets have the same dimension



$\boxtimes$.

rest: next class