② Exersice if $x \in W_1 \cap W_2$ then $7x \in W_1 \cap W_2$

③ $0 \in W_1$, $0 \in W_2 \Rightarrow 0 \in W_1 \cap W_2$

lecture 28.09.06

$W \subset V$ is a "subspace" if

A. It contains 0 and is closed under add. & multiplication by scalars $\Leftrightarrow$ B. In itself, it is a vector space with the opereation induced from V.

| ws\c | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 2 | 3 | 4 |
| 2 | 0 | 0 | 2 | 5 | 9 |
| 3 | 0 | 0 | 0 | 5 | 14 |
| 4 | 0 | 0 | 0 | 0 | 14 |

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

1 -1
1 0 -1
1 1 -1 -1
1 2 0 -2 -1
1 3 2 -2 -3 -1
1 5 0 -5 -4 -1

!hint to the match question!

continued

$x, y \in W \Rightarrow x+y \in W$

$c \in F, x \in W \Rightarrow cx \in W$

Def:

Let $(u_i) = (u_1, u_2, \cdots u_n)$ be a sequence of vectors in $V$.

A sum of the form $a_i \in F$ $\sum_{i=1}^{n} a_i u_i = a_1u_1 + a_2u_2 + a_3u_3 + \cdots\cdots + a_nu_n$

is called a "linear combination" of the $u_i$

$\underbrace{\text{span}}_{\text{could be any subset}}(u_i) :=$ the set of all possible linear combinations of the $u_i$'s.

If $S \subset V$ is any subset

$\text{span}\, S :=$ The set of all linear combinations of vectors in $S$

$= \left\{ \sum_{i=0}^{n} a_i u_i : \begin{matrix} a_i \in F \\ u_i \in S \end{matrix} \right\} \ni 0$ even if $S$ is empty. $(0! = 1)$ ??

$\sum_{i=1}^{0} x_i = 0$ so this linear combinations always contains $0$.

Thm: for any $S \subset V$, span $S$ is a subspace of $V$.

prf:

1. $0 \in \text{span}\, S$

2. $x \in \text{span}\, S$, $y \in \text{span}\, S$

$x = \sum_{1}^{n} a_i u_i \quad u_i \in S$, $\quad y = \sum_{1}^{m} b_i v_i \quad v_i \in S$

$x + y = \sum_{i=1}^{n} a_i u_i + \sum_{i=1}^{m} b_i v_i = \sum_{i=1}^{n+m} c_i w_i$ $\left\} \begin{matrix} c_i = (a_1, \cdots a_n, b_1 \cdots b_m) \\ w_i = (u_1 \cdots u_n, v_1 \cdots v_m) \end{matrix} \right.$ $w_i \in S$

3. $cx = c\sum_{i=1}^{n} a_i u_i = \sum_{i=1}^{n} (c \cdot a_i) u_i \in \text{span } S$

Example:

Let $\underset{V}{P_3(\mathbb{R})} = \{ax^3 + bx^2 + cx + d\} \subset P(\mathbb{R})$ $a,b,c,d \in \mathbb{R}$

$u_1 = x^3 - 2x^2 - 5x - 3$

$u_2 = 3x^3 - 5x^2 - 4x - 9$

$v = 2x^3 - 2x^2 + 12x - 6$

Let $W = \text{span}(u_1, u_2)$

Q: Is $v \in W$?

$v$ is in $W$ if $v = a_1 u_1 + a_2 u_2$ for some $a_1$ & $a_2 \in \mathbb{R}$

If $\exists\, a_1, a_2 \in \mathbb{R}$

$$2x^3 - 2x^2 + 12x - 6 = a_1(x^3 - 2x^2 - 5x - 3) + a_2(3x^3 - 5x^2 - 4x - 9)$$

$$= (a_1 + 3a_2)x^3 + (-2a_1 - 5a_2)x^2 + (-5a_1 - 4a_2) + (-3a_1 - 9a_2)$$

$$\Leftrightarrow \begin{aligned} 2 &= a_1 + 3a_2 \\ -2 &= -2a_1 - 5a_2 \\ 12 &= -5a_1 - 4a_2 \\ -6 &= -3a_1 - 9a_2 \end{aligned}$$

$0 = -a_1 - 2a_2$

$4 = 2a + \cancel{6}a_2$

$-2 = -2a - 5a_2$

$2 = +a_2 \Rightarrow a_2 = 2$, $a_1 = -4$

if the all the equations hold then $v \in W$