List of Theorems and Defined Terms

- ☐ **Function**
  - ☐ One-to-One
  - ☐ onto
- ☐ **Field**
  - ☐ Five properties
  - ☐ Theorems
    - ☐ Cancellation laws
    - ☐ 0, 1 unique
    - ☐ $a \times 0 = 0$
    - ☐ $(-a) \times b = a \times (-b) = -(a \times b)$, $(-a) \times (-b) = a \times b$
    - ☐ 0^{-1} doesn't exist
    - ☐ ...
- ☐ **Complex Numbers**
  - ☐ Addition, multiplication, is a field
  - ☐ The complex conjugate
    - ☐ The conjugate of the conjugate of z = z
    - ☐ ...
  - ☐ Absolute value
    - ☐ $|zw| = |z| \times |w|, |\frac{z}{w}| = \frac{|z|}{|w|}, |z+w| \leq |z| + |w|, |z| - |w| \leq |z+w|$
  - ☐ $z = e^{i\theta}$
  - ☐ Fundamental theorem of algebra (The proof in the book uses things we have not covered):
    Suppose that $p(z) = a_n z^n + a_{n-1} z^{n-1} + ... + a_0$ is a polynomial in P(C) of degree $n \geq 1$. Then p(z) has a zero.
- ☐ **Vector Space**
  - ☐ 8 Properties
  - ☐ Theorems
    - ☐ Cancellation law for vector addition
    - ☐ 0_v is unique, -x is unique
    - ☐ $0x = 0, (-a)x = -(ax) = a(-x), a0 = 0$
- ☐ **Subspaces**
  - ☐ Four properties
    - ☐ One redundant
  - ☐ Theorems
    - ☐ A subset W of V is a subspace of v iff $0 \in W$, $x + y \in W$ if $x \in W$ and $y \in W$, $cx \in W$ if $c \in F$ and $x \in W$
    - ☐ The intersection of subspaces of V is a subspace of V
    - ☐ The union of two subspaces of V is a subspace of V iff one contains the other
- ☐ **Linear Combinations**
  - ☐ Span(S)
    - ☐ Theorem: Span(S) is a subspace of V, if a subspace contains S, it contains Span(S)
    - ☐ Generating/Spanning a vector space
- ☐ **Linear Dependence / Linear Independence**
  - ☐ Trivial representation of 0
  - ☐ linear (in)dependence of $\emptyset, \{0\}, \{a\} : a \neq 0$
  - ☐ Theorems
    - ☐ let $S_1 \subseteq S_2 \subseteq V$ If S_1 is linearly dependent, then S_2 is linearly dependent. If S_2 is linearly independent, then S_1 is linearly independent.
    - ☐ Let S be a linearly independent subset of V, and let v be a vector in V not in S. Then $S \cup \{v\}$ is linearly dependent iff $v \in span(S)$
- ☐ **Basis**
  - ☐ Theorems

- ☐ let $\beta = \{u_1, u_2, ..., u_n\}$ be a subset of V. Then β is a basis for V iff all elements in V as a linear combination of vectors in β in only one way
- ☐ If V is generated by a finite set S, then some subset of S is a basis for V. Hence V has a finite basis.
- ☐ Replacement Theorem: Let V be a vector space that is generated by a set G containing exactly n vectors, and let L be a linearly independent subset of V containing exactly m vectors. Then $m \leq n$ and there exists a subset H of G containing exactly n - m vectors such that $L \cup H$ generates V
  - ☐ Let V be a vector space having a finite basis. Then every basis for V contains the same number of vectors.
  - ☐ Let V be a vector space with dimension n:
    - ☐ Any finite generating set for V contains at least n vectors, and a generating set for V that contains exactly n vectors is a basis for V.
    - ☐ Any linearly independent subset of V that contains exactly n vectors is a basis for V.
    - ☐ Every linearly independent subset of V can be extended to a basis for V.

☐ **Dimensions**

- ☐ Finite Dimensional, Infinite dimensional
- ☐ Theorems (We haven't gone over these in class yet, I don't know if they are valid subject matter for the term test)
  - ☐ Let W be a subspace of a finite-dimensional vector space V. Then W is finite-dimensional and $dim(W) \leq dim(V)$. Moreover, if $dim(W) = dim(V)$, then $V = W$.
    - ☐ If W is a subspace of a finite-dimensional vector space V, then any basis for W can be extended to a basis for V.