

The end of the proof is rushed only because the details are in the text.

* The induced orientation of ∂M is the restriction of all coord. patches of M to ∂H^k , reflected with $r : \mathbb{R}^k \ni \begin{pmatrix} x_1 \\ \vdots \\ x_k \end{pmatrix} \mapsto \begin{pmatrix} x_1 \\ \vdots \\ x_{k-1} \\ -x_k \end{pmatrix}$ if k is odd. The corresponding Dvor orientation is such that prepending the vector pointing out of M to a basis in the orientation of a point in ∂M will make that basis in the orientation of that point in M .

We didn't exactly prove this but it's extremely obvious from the preceding proof.

There are only a few results left, without much to do with Dvor orientations, their proofs are summarized:

* Let M be an oriented n -t manifold in \mathbb{R}^n

$\exists!$ bijection $\phi : \{\text{orientations of } M\} \rightarrow \{\text{continuous vectorfields on } M\}$

s.t. For orientation of M , $\forall p \in M$, $\|\phi(O)(p)\| = 1 \wedge \phi(O)(p) \in I_p(M)^\perp \wedge (\phi(O)(p), v_1, \dots, v_{n-1})$ is of the natural orientation of \mathbb{R}^n
where (v_1, \dots, v_{n-1}) are a basis of $I_p(M)$ of the orientation O

"pf": $\forall p \in M$, $I_p(M)$ is $n-1$ dimensional so, $I_p(M)^\perp$ is 1 dimensional. The fact that $\phi(O)(p)$ sits on a line & is of length 1

uniquely determines its sign, which the last condition takes care of. Given a continuous vectorfield on M that satisfies these conditions, we can apply ϕ^{-1} to it easily by just choosing at each $p \in M$, the unique orientation that satisfies the last property. As ϕ is determined by its conditions, it's unique.

For instance, the conditions define a unique value at every point, the difficulty is continuity. We can prove local continuity at every point $p \in M$ by looking at a coordinate patch $\alpha : U \ni p \mapsto V$ of p . $\exists i \in \{1, \dots, n\}$: π_i is a diffeomorphism on V . Then, at each $x \in V$, consider $(\zeta, \alpha(\zeta(x)); e_i, -\alpha(\zeta(x)); e_{n-1})$ when ζ has 1s on its i^{th} positions & 0s everywhere else.

Let $F_i : (I_x(\mathbb{R}^n))^\omega$ be the i^{th} step of the graham-schmidt process, as in, all but the n^{th} vector are unchanged while the n^{th} input is made orthogonal to the $n-1^{\text{th}}$ to n^{th} vectors. Each F_i is obviously continuous so, we can define our desired vector field to be $f(F_{n-1}, F_{n-2}, \dots, F_i(\zeta, \alpha(\zeta(x)); e_i, -\alpha(\zeta(x)); e_{n-1}))$ where f normalizes the 1^{st} input, & drops the rest. This function is continuous on V & satisfies all properties, as can be verified ■

* Let M & N be oriented compact k -manifolds

$f : \phi : M \rightarrow N$ is C^r with max rank differential, it's either orientation preserving or reversing

"pf": In the proof that a manifold is Dvor orientable (\Leftrightarrow its Munkres orientable), we showed that a parameterization

α over an open set of a manifold must agree with the manifold's orientation over the entire open set if

they agree with it at one point & must either agree or disagree at every point. This fact, along with the fact that at every point, $\phi_\#$ is either orientation preserving or reversing allows us to prove this by considering

$\{\phi \circ \alpha_i\}$, where α_i are coordinate patches on M . The idea is in the picture ■

