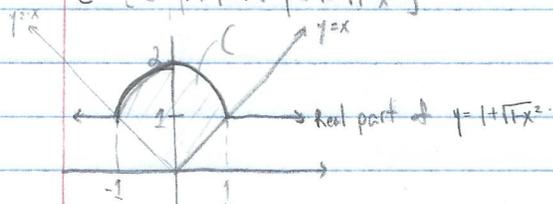


Problem A.

$$C = \{ (x, y) : |x| \leq 1 \leq y \leq 1 + \sqrt{1-x^2} \}$$



$$C = 2 \left[\int_0^1 x \, dx + \int_0^1 (1 + \sqrt{1-x^2}) - 1 \, dx \right]$$

$$= 2 \left[\frac{1}{2} x^2 \Big|_0^1 + \int_0^1 \sqrt{1-x^2} \, dx \right]$$

$$= 2 \left[\frac{1}{2} x^2 \Big|_0^1 + \frac{1}{2} (\sqrt{1-x^2} + \sin^{-1}(x)) \Big|_0^1 \right]$$

$$= 2 \left[\left(\frac{1}{2} - 0 \right) + \left(\frac{1}{2} \sin^{-1}(1) - \frac{1}{2} \sin^{-1}(0) \right) \right]$$

$$= 1 + \sin^{-1}(1) - \sin^{-1}(0)$$

$$= 1 + \sin^{-1}(1)$$

9

13.4. Given $f: S \cup S_2$ bounded, and $\int_{S_1} f, \int_{S_2} f$ exists,

$S_1 \cap S_2$ is integrable by f .

and taking $S_1 - S_2 \cup S_1 \cap S_2 = S_1$, but $(S_1 - S_2) \cap (S_1 \cap S_2) = \emptyset$

We know S_1 and $S_1 \cap S_2$ is integrable

meaning the disc-set $D(S_1)$ and $D(S_1 \cap S_2)$ is of measure zero

and because $(S_1 - S_2) \cup (S_1 \cap S_2) = S_1$

$$D(S_1 - S_2) \cup D(S_1 \cap S_2) = D(S_1)$$

and by Thm 11.1(b) $D(S_1 - S_2)$ is of measure zero

so $S_1 - S_2$ is of measure zero, $\int_{S_1 - S_2} f$ exists.

The rest follows by Thm 13.4.

$$\int_{S_1} f = \int_{S_1 - S_2} f + \int_{S_1 \cap S_2} f$$

$$\int_{S_1 - S_2} f = \int_{S_1} f - \int_{S_1 \cap S_2} f$$

Q.E.D.

14.4 $\{1\} \rightarrow$ Bounded, Measure zero, $\text{Bd}(\{1\}) = \{1\} \rightarrow$ measure zero so Rectifiable ✓

$\mathbb{Q} \cap [0,1] \rightarrow$ Bounded, Measure zero (proven in a previous problem set) ✓

$\text{Bd}(\mathbb{Q} \cap [0,1]) = [0,1] \rightarrow$ not measure zero so, not Rectifiable. 10

14.8. $\int_{y \in B} v(S_y) = \int_{y \in B} \int_{x \in A} I_{S_y}$

However given some y_0 , $\int S_{y_0} = \int S(y_0)$ ✓

so $\int_{y \in B} \int_{x \in A} I_{S_{y_0}} = \int_{y \in B} \int_{x \in A} I_S$

$\int_{y \in B} \int_{x \in A} I_S = \int_{\{x,y\} \in A \times B} I_S = v(S)$

$\therefore \int_{y \in B} v(S_y) = v(S)$