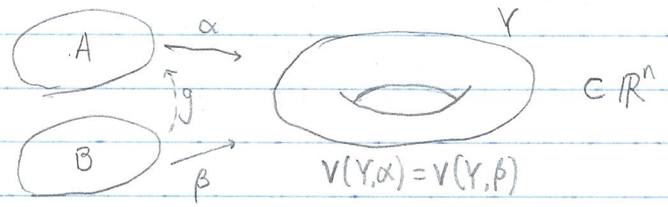
Jan 18th **Riddle** On $\mathbb{Z}\times\mathbb{Z}$, a visible roach R starts at $(0,0)$ and once a minute jumps to the northeast, up to a distance of 10. Meanwhile, an exterminator E can poison one grid point per minute, away from R. Can E trap R?

Read Along Section 23&24.

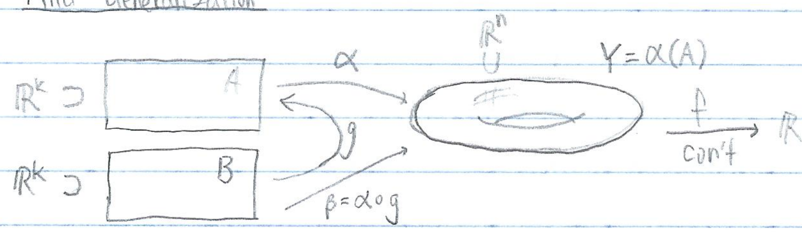
- $V(Y) = V(Y,\alpha) := \int_A V(D\alpha) := \int_A |\det D\alpha^T D\alpha|^{1/2}$



$V(Y,\alpha) = V(Y,\beta)$

Precisely, if $g: B \to A$ is a diffeomorphism of open sets in $\mathbb{R}^k$, and $\alpha: A \to \mathbb{R}^n$ is a manifold, set $\beta = \alpha\circ g$ and then $\alpha(A) = \beta(B) = Y$ and $V(Y,\alpha) = V(Y,\beta)$

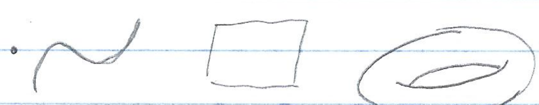
Mild Generalization



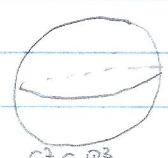
Def $\int_Y f\, dV = \int_A (f\circ\alpha)\, V(D\alpha) \overset{\text{Thm}}{=} \int_B (f\circ\beta)\, V(D\beta)$

- $\int_M d\omega = \int_{\partial M} \omega$ $\quad$ M - A nice & smooth k-dim subset of $\mathbb{R}^n$

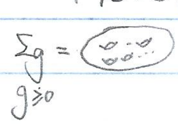
Examples:

- 

$T^2, S^1\times S^1, \Sigma, \subset \mathbb{R}^3$



$S^2 \subset \mathbb{R}^3$

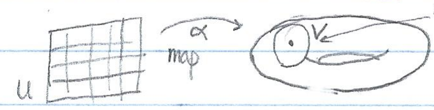


Klein bottle $\subset \mathbb{R}^4$

$O(n) = \{A \in M_{n\times n} : A^T A = I\} \subset \mathbb{R}^{n^2}$

$O(2) = S^1 \cup S^1 \subset \mathbb{R}^4$

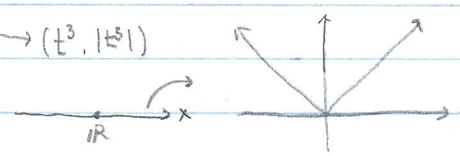
Def A k-dim manifold (without boundary, of class C^r, $r \geq 1$) in $\mathbb{R}^n$, is a subset $M \subset \mathbb{R}^n$ s.t. each $P \in M$ has an open nbd V s.t. there is an open $U \subset \mathbb{R}^k$ and a C^r homeomorphism $\alpha : U \to V$, whose differential $D\alpha(x)$ is of rank k for every $x \in U$



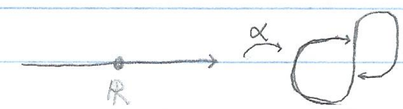
<u>Non-examples</u> • $\alpha : \mathbb{R} \to \mathbb{R}^2$ by $t \mapsto (t^3, t^2)$

α is a homeomorphism. $D\alpha = (3t^2, 2t)$, $D\alpha$ is rank 0 at $t=0$

• $t \longmapsto (t^3, |t^3|)$



•



$D\alpha \neq 0$ but non-homeomorphic.