

Q Let

$$A = \mathbb{R}^1 \ni x \mapsto T_x \in$$

Specific example for 27.(b).

$$V = \mathbb{R}^2.$$

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \text{ W along } W_1.$$

$$W = \{(x, 0) \mid x \in \mathbb{R}\}, \beta_1 = \{(1, 0)\}$$

$$W_1 = \{(0, y) \mid y \in \mathbb{R}\}, \beta_2 = \{(0, 1)\}$$

$$\text{Then } W \cap W_1 = \{0\} \quad W + W_1 = \mathbb{R}^2 \Rightarrow W \oplus W_1 = \mathbb{R}^2$$

$$v_0 = (1, 0) \quad ((x, y), x, y \in \mathbb{R})$$

$$\Rightarrow \beta_3 = \{(1, 1)\} \Rightarrow W_2 = \{(z, z) \mid z \in \mathbb{R}\}$$

$$W \cap W_2 = \{0\} \quad W + W_2 = \mathbb{R}^2 \Rightarrow W \oplus W_2 = \mathbb{R}^2.$$

$$((x+z, z), x, z \in \mathbb{R})$$

$$U: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \text{ W along } W_2.$$

$$\text{Hence. } T(x, y) = (x, 0), \quad x, y \in \mathbb{R} \quad \text{G}$$

$$U(x, y) = (x-y, 0), \quad x, y \in \mathbb{R}.$$

P85

$$5.(b) \quad f(x) = 1 \quad f'(x) = 0 \quad f''(x) = 0$$

$$f(x) = x \quad f'(x) = 1 \quad f''(x) = 0$$

$$f(x) = x^2 \quad f'(x) = 2x \quad f''(x) = 2$$

① Let  
 $A = \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$

Prob  
16. Prob

$$T(1) = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$$

$$T(x) = \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$$

$$T(x^2) = \begin{pmatrix} 0 & 2 \\ 0 & 2 \end{pmatrix}$$

Suppose  $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$

$$\beta = \{\beta_1, \beta_2, \beta_3\}$$

$$\Rightarrow T(\beta_1) = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} = \sum_{i=1}^4 a_{i1} \alpha_i = a_{11} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + a_{21} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + a_{31} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + a_{41} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow a_{11} = a_{31} = a_{41} = 0 \quad a_{21} = 2.$$

$$T(\beta_2) = \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix} = \sum_{i=1}^4 a_{i2} \alpha_i = a_{12} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + a_{22} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + a_{32} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + a_{42} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow a_{32} = a_{42} = 0 \quad a_{12} = 1 \quad a_{22} = 2$$

$$T(\beta_3) = \begin{pmatrix} 0 & 2 \\ 0 & 2 \end{pmatrix} = \sum_{i=1}^4 a_{i3} \alpha_i = a_{13} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + a_{23} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + a_{33} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + a_{43} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow a_{13} = a_{33} = 0 \quad a_{23} = a_{43} = 2$$

$$\Rightarrow [T]_{\beta}^{\alpha} = \begin{pmatrix} 0 & 1 & 0 \\ 2 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

6

Q Let

P86  
16. Proof:

We just take an arbitrary  $\beta = \{v_1, v_2, \dots, v_n\}$   
which is an ordered basis of  $V$ .

We suppose that  $T(v_i) = w_i \quad w_i \in W \quad i=1, 2, \dots, n$

① When  $\text{rank}(T) < \dim W$

$$\because R(T) = \text{span}(T(\beta))$$

$$= \text{span}(T(v_1), T(v_2), \dots, T(v_n))$$

$$= \text{span}(w_1, w_2, \dots, w_n)$$

and  $\dim(R(T)) < \dim W$

$\exists i \leq n$ , s.t.  $w_i$  is a linear combination of others.

Then we remove  $w_i$  from  $\{w_1, w_2, \dots, w_n\}$  until there does not exist  $i \leq n$  s.t.  $w_i$  is a linear combination of others. And we will get a new set  $\{w_1, w_2, \dots, w_m\}$   $m < n$  after adjusting the order. i.e.  $\{w_1, w_2, \dots, w_m\}$  is a linearly independent subset of  $W$ .

We extend  $\{w_1, w_2, \dots, w_m\}$  to a basis of  $W$  which means we will get  $\{w_1, w_2, \dots, w_m, w_{m+1}, \dots, w_n\}$ .

Now we adjust the order of  $v_i$  in  $\beta$ , such that  $T(v_i) = w_i \quad i \leq m$ .

$$T(v_j) = w_i \quad i \leq m \quad T(v_j) = \sum_{i=1}^m d_{ij} w_i \quad i \leq m \quad d_{ij} \neq 0 \\ j > m$$