Example 3

ie any set,

LET S BE A SET $\left\{\begin{matrix} CAT \\ COW \\ FRISBEE \end{matrix}\right\}$ Subexample

LET F BE A FIELD

e Function

$\mathcal{F}(S, F_{\text{field}}) = \{ \varphi : S \longrightarrow F \}$ (The set of all functions that map S to F)

$\mathcal{F}(S, \mathbb{R}) = \left\{ \begin{pmatrix} CAT \to -\pi \\ FRISBEE \to 2.7 \\ COW \to 0 \end{pmatrix}, \begin{pmatrix} COW \to 1{,}000{,}000 \\ Frisbee \to 3 \\ CAT \to 0 \end{pmatrix}, \ldots \underset{0_{\mathcal{F}}}{\begin{pmatrix} COW \to 0 \\ Frisb \to 0 \\ CAT \to 0 \end{pmatrix}} \right\}$

each list is 1 particular function

- $0_{\mathcal{F}(S,F)}(\sigma) = 0_F$

$\sigma \in S$

ie σ can be CAT, or FRISBEE ETC.

- $\varphi_1, \varphi_2 \in \mathcal{F}(S, F)$

both are elements in the field.

Addition: $(\varphi_1 + \varphi_2)(\sigma) := \varphi_1(\sigma) +_F \varphi_2(\sigma)$.

$\sigma \in S$

- $(c\varphi)(\sigma) := c(\varphi(\sigma)) \in F.$

in our sub example ie cat etc.

$\varphi_1 + \varphi_2 = \begin{pmatrix} COW \to 0 + 10^6 = 10^6 \\ FRISBEE \to 2.7 + 3 = 5.7 \\ CAT \to -\pi + 0 = -\pi \end{pmatrix}$

$2\varphi_1 = \begin{pmatrix} COW \to 2(0) = 0 \\ CAT \to 2(-\pi) = -2\pi \\ FRISB \to 2(2.7) = 5.4. \end{pmatrix}$

CLAIM: $\mathcal{F}(S,F)$ is a V.S.

VS2 $\varphi_1+(\varphi_2+\varphi_3) = (\varphi_1+\varphi_2)+\varphi_3$?

Let $\sigma \in S$

L.S. $[\varphi_1 + (\varphi_2+\varphi_3)](\sigma) = \varphi_1(\sigma) +_F (\varphi_2 +_F \varphi_3)(\sigma)$

$= \varphi_1(\sigma) +_F (\varphi_2(\sigma) +_F \varphi_3(\sigma))$

R.S. $[(\varphi_1+\varphi_2)+\varphi_3](\sigma)$ ----- $= (\varphi_1(\sigma)+\varphi_2(\sigma)) +_F \varphi_3(\sigma)$.

L.S = R.S. By F_2 (Associative Law).

sub-example. $S = \{1, 2, \dots n\}$

$$\mathcal{F}(S,F) = \left\{\begin{pmatrix} 1 \to \text{in } F \\ 2 \to \text{in } F \\ n \to \text{in } F \end{pmatrix}\right\} \leftrightarrow \left\{\begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}\right\} a_i \in F$$

$= F^n$.

[o This example contains the Example #1, Also you can check that this example also contains the matrix example]

Example 4. $P_n(F) =$ "polynomials of degree n over F"

$$= \{a_n x^n + a_{n-1} x^{n-1} + \dots a_1 x + a_0 : a_i \in F\}$$

$0 = 0x^n + \dots 0x^0$