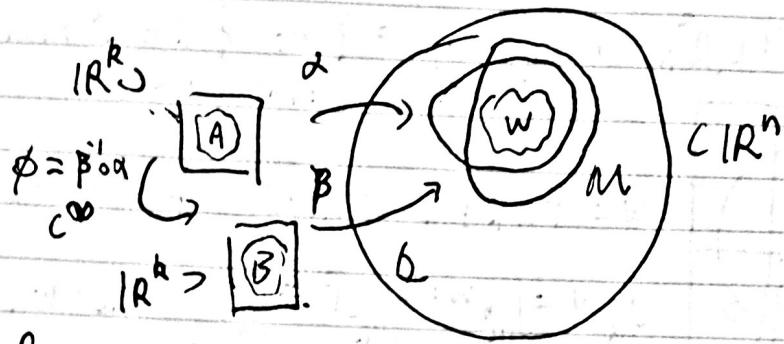


15th Wed. March. Hour 063.

Read along: 33-35

$M^k \subset \mathbb{R}^n$, $w \in \Omega^{\text{top}}(M)$; $\text{Supp}(w) \subset U = \text{Im } \alpha$.
 $\alpha: A \subset \mathbb{R}^k \rightarrow M$, it's a patch



$$\int_M w = \int_A \alpha^* w = \int_A f, \text{ where } \alpha^* w = f dx_k$$

Claim Makes sense up to $\pm \frac{1}{2}$

Precisely, $\int_M w = \pm \int_M w$ provided $\text{Supp } w \subset V = \text{Im } \beta$

next battle $\beta: B \subset \mathbb{R}^k \rightarrow M$, a patch

provided also $U \cap V$ is connected (path)

PF: $\int_M^B w = \int_B g$, where $\beta^* w = g dx_k$

$$\int_M^B w = \int_{A := \alpha^{-1}(U \cap V)} \alpha^* w \stackrel{\alpha = \beta \circ \phi}{=} \int_{A'} (\beta \circ \phi)^* w = \int_{A'} \phi^* (\beta^* w) = \int_{A'} \phi^* (g dx_k)$$

On U connected set, $\det(D\phi)$

is either uniformly + or uniformly -

$$= \int (g \circ \phi) \cdot \det(D\phi) dx_k = \int_{A'} (g \circ \phi) \det D\phi = \pm \int (g \circ \phi) / \det D\phi$$

$$= \pm \int g = \pm \int_B g = \pm \int_M^B w$$

$$B' = \beta^{-1}(U \cap V)$$

Oriented manifolds / oriented vector spaces

Definition: An orientation \mathcal{O} on a f.d. v.s. V is a choice basis for V , regarded up to positive det changes.

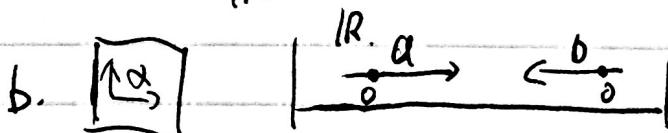
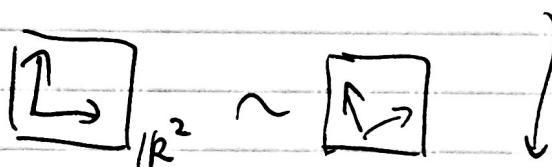
$(v_1, \dots, v_n) \xrightarrow{\sim} (u_1, \dots, u_n)$ if

$$\det C_{v_i}^u > 0, \text{ where } C_{v_i}^u = (c_{ij}), \text{ where } u_i = \sum c_{ij} v_j$$

An oriented v.s. is a f.d. v.s. along with a choice of an orientation on it.

Ex: 1.a / 1.b ~~1.b~~ right handed, left handed

2.a / 2.b



Claim: Every f.d. v.s. V has exactly 2 distinct orientation

Pf: for any basis of $V: v_1, \dots, v_n$, then $\mathcal{O}_1 = (v_1, \dots, v_n)$ is an orientation of V . and

$\mathcal{O}_2 = (-v_1, v_2, \dots, v_n)$ a second orientation of V .

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \det = -1$$

Suppose u_1, \dots, u_n is another basis if $\det C_{v_1, \dots, v_n}^{u_1, \dots, u_n} > 0$
 u_1, \dots, u_n represents the same orientation as ~~v_1, \dots, v_n~~ v_1, \dots, v_n

otherwise $C_{v_1, \dots, v_n}^{u_1, \dots, u_n} = C_{v_1, \dots, v_n}^{u_1, \dots, u_n} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

C_1

C_2

E

Similary, Suppose O is given by some basis

This orientation is "reversed". (switched to the other)

if 1. You negate any of the ~~the~~ vectors in the basis

2. You swap two vector $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$