So pick j s.t. $a_j \neq 0$, by the observation before

(*) $u_j \in \text{span}(\{u_i : i \neq j\} \cup L)$

Let $R = R' \cup \{u_j\}$ then $|R| = n+1$

<u>claim</u> $\text{span}((G \setminus R) \cup L) = V$

Indead it is enough to show that

$(G \setminus R') \cup L' \subset \text{span}((G \setminus R) \cup L)$

but clearly every element of $(G \setminus R')$ is in RHS maybe except u_j, $u_j \in$ RHS by *.

Corollaries:

1. If V has a finite basis, the every other basis is also finite and has the same number of elements $\Rightarrow \dim V$ makes sense

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2. Assume $\dim V = n$, then 1a. if G generates V, then $|G| \geq n$ & if $|G| = n$, then it is a basis.

b. If L is a lin indep subset. then $|L| \leq n$, If further $|L| = n$, then L is a basis and furthermore, if $|L| < n$ then L can be extended to a basis of V

<u>PF of a</u> Let $\beta = \{u_1, \cdots u_n\}$ of V

by replacement, find $R \subset G$ s.t. $|R| = n$ & s.t.

$\operatorname{span}((G \setminus R) \cup \beta) = V$

$\Rightarrow |G| \geq n$ & if $|G| = n$.

then it is lin indep. Indeed if it wasn't then there is some non-trivial LC equal to 0 in G, so an element of G is a LC of the others, so G has a strict subset that still generates, but that can't be, because every generating set has $\geq n$ elements

<u>Pf of b</u>

Assume $|\beta| = n$ is a basis, L is lin indep., then by replacement using $G = \beta$, $|L| \leq |\beta| = n$, If $|L| = n$, then by replacement $\exists R \subset G$ w/ $|R| = n$ $\begin{pmatrix} \text{so} \\ G = R \end{pmatrix}$ s.t.

$\text{span}((G \setminus R) \cup L) = V$

So $\text{span}(\emptyset \cup L) = V$

So, L generates V

So L is a basis

Finally, if $|L| < n$

$\text{span}\, L \subsetneq V$ pick

$u_1 \in V \setminus \text{span}(L)$