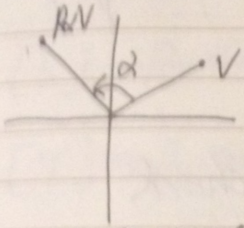
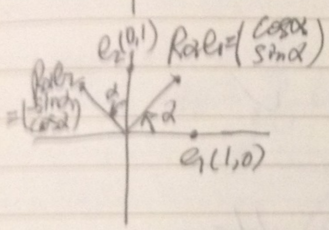
3. $R_\alpha : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$



$$\beta = \gamma = \left(\underset{e_1}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}, \underset{e_2}{\begin{pmatrix} 0 \\ 1 \end{pmatrix}} \right)$$



$$R_\alpha e_1 = (\cos\alpha)(e_1) + (\sin\alpha)(e_2)$$

$$R_\alpha e_2 = (-\sin\alpha) e_1 + (\cos\alpha) e_2$$

$$[R_\alpha] = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

4. $A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$ $T_A : F^n \longrightarrow F^m$

$$T_A \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ \vdots \\ a_{m1}x_1 + \cdots\cdots + a_{mn}x_n \end{pmatrix}$$

$$[T_A]_{(e_i)}^{(e_i)} = \left(\begin{matrix} a_{11} \\ \vdots \\ a_{m1} \end{matrix} \middle| \begin{matrix} a_{12} \\ \vdots \\ a_{m2} \end{matrix} \middle| \cdots \middle| \begin{matrix} a_{1n} \\ \vdots \\ a_{mn} \end{matrix} \right) = A$$

$$T_A \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} a_{11} \\ \vdots \\ a_{m1} \end{pmatrix} = a_{11} e_1 + \cdots + a_{m1} e_m.$$

The remaining issue is the linearly of

$$T \to [T]_\beta^\gamma$$

meaning we need to check

0. $0 \to [0]_\beta^\gamma = 0$ ✓

1. If $T, S : V \to W$ then

$$[T+S]_\beta^\gamma \stackrel{?}{=} [T]_\beta^\gamma + [S]_\beta^\gamma \quad \square$$

2. $[cT]_\beta^\gamma \stackrel{?}{=} c \cdot [T]_\beta^\gamma \quad \square$

$$\begin{tikzcd}
U \arrow[r, "S", "B={[S]}"'] & V \arrow[r, "T", "A={[T]}"'] & W \\
\alpha=\{u_1, \cdots u_p\} & \beta=\{v_1 \cdots v_n\} & \gamma=\{w_1, \cdots w_m\} \\
\dim p \arrow[rr, bend right, "T \circ S", "C"'] & n & m
\end{tikzcd}$$

If we know A & B can we figure C?

$$[S] = B \Leftrightarrow Su_i = \sum_{j=1}^{n} b_{ji} v_j$$

$$[T] = A \Leftrightarrow Tv_j = \sum_{k=1}^{m} a_{kj} w_k$$

$$\Rightarrow Cu_i = \underbrace{\sum_{j=1}^{n} b_{ji} \sum_{k=1}^{m} a_{kj} w_k}_{\text{coeff of } w_k} = \underbrace{\sum_{k=1}^{m} c_{ki} w_k}_{\text{coeff of } w_k}$$

$$\sum_{j=1}^{n} a_{kj} b_{ji} = c_{ki}$$

<u>Def</u> Given $A \in M_{m\times n}$ $B \in M_{n\times p}$

Define a new matrix called

$C = A \cdot B \in M_{m\times p}$ by setting

$$c_{ki} = \sum_{j=1}^{n} a_{kj} b_{ji}$$

<u>Thm</u> In our situation

$$\underset{m\times p}{[T\circ S]_{\alpha}^{\gamma}} = \underset{m\times n}{[T]_{\beta}^{\gamma}} \cdot \underset{n\times p}{[S]_{\alpha}^{\beta}}$$