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$\Rightarrow k \le n$

So k is finite, Let γ be a lin indep subset of W w/ $|\gamma| = k$

Claim

β is a basis, β is lin indep

β generate W

If not, then pick $u \in W \setminus \text{span}(\beta)$.

and the $\beta \cup \{u\}$ is lin.indep and contained in W contradicting the maximality of β

If $\dim W = \dim V$ then $V = W$

(because if β is a basis of W, then it is lin indep in V, of size equal to $\dim(V)$

So β is a basis of V. So $V = \text{span}(\beta) = W$)

If $\dim W < \dim V$, then every basis of W can be extended to a basis of V

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PF Let β be a basis of W

γ be a basis of V

use replacement to find $R \subset \gamma$

s.t. $(\gamma \setminus R) \cup \beta$ generates V & $|R| = |\beta|$

Now $|(\gamma \setminus R) \cup \beta| = |\gamma| = \dim V$

So $(\gamma \setminus R) \cup \beta$ is a basis of V.

which contains β

$\square$

Term test ends here

Thm Every v.s. has a basis

Example $\mathbb{R}$ is a v.s over $\mathbb{Q}$

basis

Application "Lagrange Interpolation Thm"

Suppose $x_1, \cdots, x_{n+1} \in \mathbb{R}, F$ all different from each other

Suppose $y_1 \cdots y_{n+1} \in \mathbb{R}$, F abitrary

Does there exist a poly $P \in P_n(\mathbb{R})$ s.t

$\forall i \quad P(x_i) = y_i$

Who cares?

1. Scientists
2. computer graphes programs.

$$\tilde{P}_j(x) = \prod_{i \neq j} (x - x_i)$$

$$\tilde{P}_j(x_k) = \prod_{i \neq j} (x_k - x_i) = \begin{cases} 0 & k \neq j \\ \prod_{i \neq j} (x_j - x_i) \neq 0 & k = j \end{cases}$$

set

$$P_j(x) = \frac{\tilde{P}_j(x)}{\tilde{P}_j(x_j)} = \frac{\prod_{i \neq j} (x - x_i)}{\prod_{i \neq j} (x_j - x_i)}$$

$$P_j(x_k) = \begin{cases} 0 & k \neq j \\ 1 & k = j \end{cases}$$

New set

$$P(x) = \sum_{i=1}^{n+1} y_i P_i(x) \quad \text{Q.E.}$$