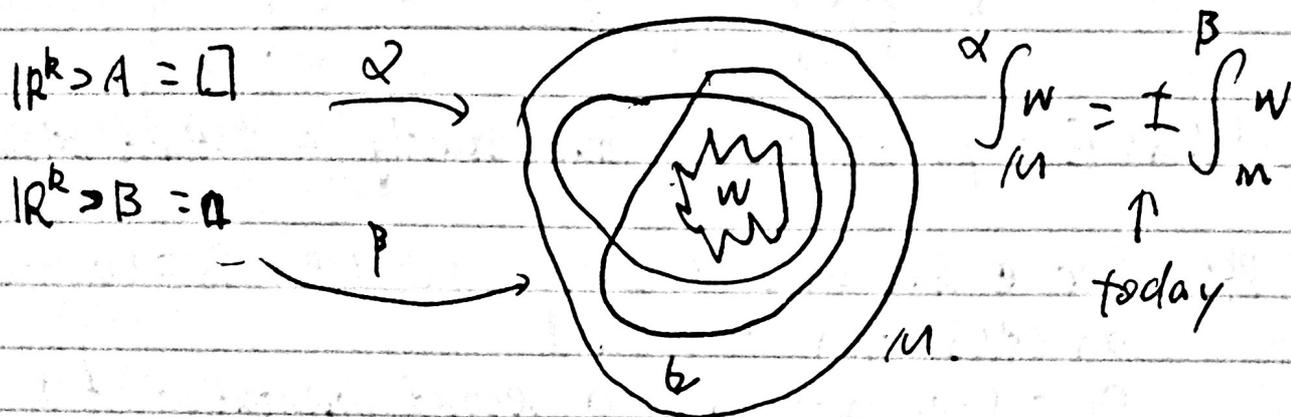


17th Fri March Hour 064

Read: 33~35

Important! No class Monday 24th. Make up on Thursday 31st
5pm. MP134. Video will be available



Def: An orientation \mathcal{O} on a f.d. v.s. V is a choice of an ordered basis for V , regarded up to a positive-det change of basis

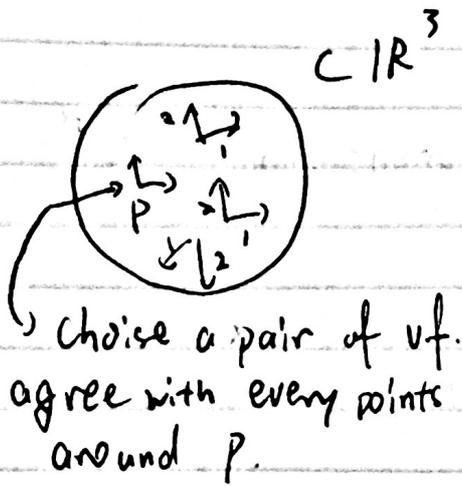
$(v_1, \dots, v_n) \sim (u_1, \dots, u_n)$ if $\det(C_{v,u}) > 0$

Every f.d. v.s. has precisely two orientations

Dmitry: what are they called?

$1 = \sqrt{i} = \sqrt{-1} = \sqrt{-1} \cdot \sqrt{-1} = i \cdot i = -1$

Def. An orientation O on a Mfd M is a cont. choice of an orientation O_x on $T_x M$ for every $x \in M$



cont: Every $P \in M$ has a nbd W with cont. v.f. u_1, \dots, u_k defined on W such that $\forall x \in W$ $(u_1(x), u_2(x), \dots, u_k(x)) \sim O_x$



1. \curvearrowright

2. rotating

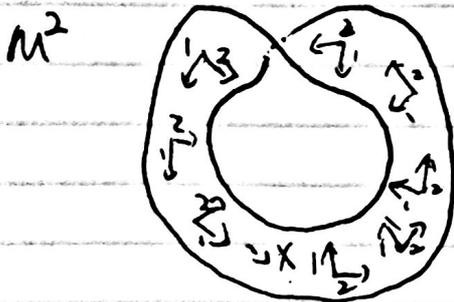


every v.f. on S^2 has at least one zero

\uparrow
orientable

\downarrow
choose an orientation \Rightarrow 'oriented'

"You cannot comb a coconut"



not original 1-2

not orientable

Properties: If $M^k \subset \mathbb{R}^{k+1}$ is connected and \mathbb{R}^{k+1} is oriented, then there is a ~~bijection~~ bijection

$\left\{ \begin{array}{l} \text{orientations} \\ \text{of } M \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} \text{cont. varying choice of unit} \\ \text{normal vectors to } M \text{ in } \mathbb{R}^{k+1} \end{array} \right\}$

So $\forall p \in M, n(p) \in T_p \mathbb{R}^{k+1}$ such that:

1. $n(p) \perp T_p M$

2. $\|n(p)\| = 1$

3. $P \mapsto n(p)$ is cont.

PF. orientation of M : Given $p \in M$, have vectors $(u_1, \dots, u_k) \sim \mathcal{O}_p$. find $v \in T_p \mathbb{R}^{k+1}$ s.t.

1. $v \perp u_i$
2. $\|v\| = 1$
3. $(u_1, \dots, u_k, v) \sim$ given orientation of \mathbb{R}^{k+1}

U band: $M^2 \subset \mathbb{R}^3 \subset \mathbb{R}^4$

claim: If M^k is orientable, then so is ∂M

Directive orient ∂M so that when prepending (add in the beginning) the orientation of ∂M , get the orientation of M .

