

MA1240

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Principle If you have n random cancelling quantities, about $\frac{n}{2}$ of these won't cancel

T^n

If $Tv = \lambda v \rightsquigarrow T^n v = \lambda^n v$.

λ is an eigenvalue

$0 \neq v$ is an eigenvector.

seek a basis made of eigenvectors

$A v = \lambda v \Leftrightarrow (A - \lambda I) v = 0 \Leftrightarrow A - \lambda I$ non-inv.
 $\Leftrightarrow \det(A - \lambda I) = 0.$

change of Basis. $T: V \rightarrow W$

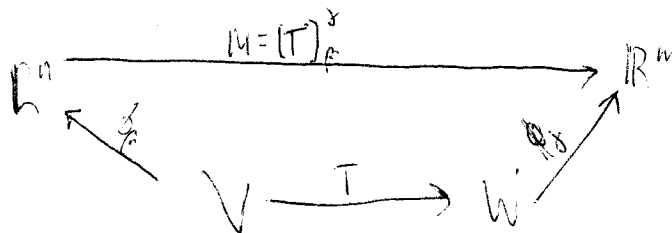
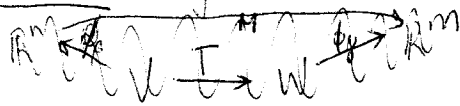
Suppose you know how to write T relative to a basis β of V and γ of W .

How do we write T relative to some other basis β' of V and γ' of W ?

$[T]_{\beta}^{\gamma} \rightsquigarrow [T]_{\beta'}^{\gamma'}$
 how?

Solution: By chase around a "commutative diagram."

$\beta = (v_1, \dots, v_n)$
 basis of V
 $\phi_{\beta}: \sum x_i v_i \mapsto \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$



First col of $M = M \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \phi_{\gamma} \left(T \left(\phi_{\beta}^{-1} \left(\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \right) \right) \right) = T v_1$
 $=$ first col of $[T]_{\beta}^{\gamma}$