MA1240 Nov 3, 9-11

Principle If you have n random cancelling quantities, about $\sqrt{n}$ of these won't cancel.

T^n

If $\underbrace{Tv = \lambda v}_{\substack{\lambda \text{ is an eigenvalue} \\ 0 \neq v \text{ is an eigenvector}}} \rightsquigarrow T^n v\ \lambda^n v.$ seek a basis made of eigenvectors.

$$Av = \lambda v \Leftrightarrow (A - \lambda I)v = 0 \Leftrightarrow A - \lambda I \text{ non-inv.}$$
$$\Leftrightarrow \det(A - \lambda I) = 0.$$

<u>Change of Basis.</u> $T: V \to W$

Suppose you know how to write T relative to a basis β of V and γ of W.
How do we write T relative to some other basis β' of V and γ' of W?

$$[T]_\beta^\gamma \underset{\text{how?}}{\rightsquigarrow} [T]_{\beta'}^{\gamma'}$$

<u>Solution</u>: By chase around a "Commutative diagram."

$\beta = (v_1, \dots v_n)$ basis of V

$\phi_\beta : \sum \alpha_i v_i \mapsto \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix}$

$$\begin{tikzcd} \mathbb{R}^n \arrow[rrr, "M = {[T]}_\beta^\gamma"] & & & \mathbb{R}^m \\ & V \arrow[ul, "\phi_\beta"] \arrow[r, "T"] & W \arrow[ur, "\phi_\gamma"] & \end{tikzcd}$$

$$\begin{array}{l}\text{First} \\ \text{col} \\ \text{of} \\ M\end{array} = M \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \underbrace{\phi_\gamma(T(\underbrace{\phi_\beta^{-1}\begin{pmatrix} 1 \\ \vdots \\ 0 \end{pmatrix}}_{v_1}}_{Tv_1} = Tv_1 \text{ in } \dots$$
$$= \text{first col of } [T]_\beta^\gamma$$