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$A=(a_{ij})$ $B=(b_{ij})$

$C=A+B=(c_{ij})$

then $c_{ij}=a_{ij}+b_{ij}$

$7\begin{pmatrix}1&2\\3&4\end{pmatrix}=\begin{pmatrix}7&14\\21&28\end{pmatrix}$

claim These defns satisfy VS1–VS8.

e.g. 3 Let S be a set $S_1=\begin{Bmatrix}\text{cat}\\\text{cow}\\\text{bee}\end{Bmatrix}$

Let F be a field

$\mathcal{F}(S,F):=\{\varphi: S\to F\}$

function field

$\mathcal{F}(S_1,\mathbb{R})=$

$\left\{\begin{pmatrix}\text{cat}\to -\pi\\\text{cow}\to 0\\\text{bee}\to 2.7\end{pmatrix}, \begin{pmatrix}\text{cat}\to 10^{100}\\\text{cow}\to 0\\\text{bee}\to 0\end{pmatrix} \text{------} \begin{pmatrix}\text{cat}\to 0\\\text{cow}\to 0\\\text{bee}\to 0\end{pmatrix}\right\}$

0v

one function

$0_{\mathcal{F}(S,F)}(\sigma)$

$\sigma \in S \quad = 0_F$

$\varphi_1, \varphi_2 \in \mathcal{F}(S,F)$

$(\varphi_1 + \varphi_2)(\sigma) := \varphi_1(\sigma) +_F \varphi_2(\sigma)$ F

$\sigma \in S$

$c \in F \quad \varphi \in V$

$(c\varphi)(\sigma) := c(\varphi(\sigma))$.

Claim $\mathcal{F}(S,F)$ is indeed a V.S.

V.S. 2 $\quad \varphi_1 + (\varphi_2 + \varphi_3) = (\varphi_1 + \varphi_2) + \varphi_3$

Let $\sigma \in S$

$[\varphi_1 + (\varphi_2 + \varphi_3)](\sigma)$

$= \varphi_1(\sigma) +_F (\varphi_2 +_V \varphi_3)(\sigma)$.

$= \varphi_1(\sigma) + (\varphi_2(\sigma) + \varphi_3(\sigma))$

$[(\varphi_1 + \varphi_2) + \varphi_3](\sigma) \quad \| \; F_2$

$= \ldots = [\varphi_1(\sigma) + \varphi_2(\sigma)] + \varphi_3(\sigma)$

sub-eg $S = \{1, 2, \cdots, n\}$

$$\mathcal{F}(S, F) = \left\{ \begin{pmatrix} 1 \to \text{in } F \\ 2 \to \text{in } F \\ \vdots \\ n \to \text{in } F \end{pmatrix} \right\} \leftrightarrow \left\{ \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \middle| a_i \in F \right\}$$

$$= F^n.$$

eg4 $P_n(F) =$ "polynomials of degree n over F"

$$= \{ a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \mid a_i \in F \}.$$

$$0 := 0x^n + \cdots + 0x^0 = 0.$$

$$(2x^2 - 3x + 7) + (x^4 - x) = x^4 + 0x^3 + 2x^2 - 4x + 7$$

$$2(-x^5 + 240) = -2x^5 + 480.$$

$$P(F) = \left\{ \begin{matrix} \text{polynomial} \\ \text{of any degree} \end{matrix} \right\} = \{ x^{107} - 7,\ x^{2014} + \pi x^{100}, \cdots \}$$