1. $|X \cup Y| = |X| + |Y|$ if $X \cap Y = \phi$

2. $|X \times Y| = |X||Y|$

3. $f : X \longrightarrow Y$ M-to-1 & onto

$$|X| = M|Y|$$

<u>words</u>: $A = \{a_1, \cdots, a_n\}$ A^k = words of length k in A

$$|A^k| = n^k \qquad \underbrace{\underline{n}\,|\,\underline{n}\,|\,\cdots\,|\,\underline{n}}_{k \text{ times}} = n^k$$

Permutations

Same but no letter repetitions

$$n(n-1)(n-2)\cdots(n-k+1) = \frac{n!}{(n-k)!} = P_k^n$$

Arrangements

5 a's. 7 b's. 13 c's. How many words?

$$\frac{(5+7+13)!}{5!\,7!\,13!} = \binom{25}{5,\,7,\,13}$$

Combinations

of ways choosing k of n objects.

= arrangements of k 1's & $(n-k)$ 0's

$$C_k^n = \binom{n}{k,\,n-k} = \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

Candy-bar principle / Distributions

How many ways to distribute n identical candies among k kids:

Combinations of $\underbrace{C\ C\ C \cdots C}_{n}\ \underbrace{B\ B\ B \cdots B}_{k-1}$

$$\binom{n+k-1}{k-1} = \frac{(n+k-1)!}{n!\,(k-1)!} = D_k^n$$

Example

① $\dfrac{6!}{2!\,3!}$

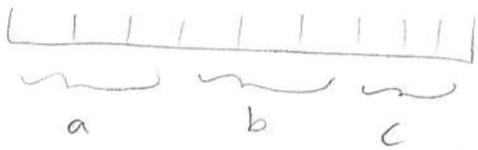
② $D_3^6 = \binom{8}{2}$

6 of $\left\{ \begin{array}{l} H \\ M \\ T \end{array} \right.$

Candies: blank order forms

kids: H, M, T

③



ABAACCCBB

$\dfrac{(3!)^3\,3!}{9!}$

$\dfrac{3!}{9!/(3!)^3}$

⑤ aaaa bbbb cccc dddd

$\cancel{\binom{10}{2}\binom{8}{2}\binom{6}{2}\binom{4}{2}\cdot 4\cdot 4}$ ← wrong