

IF Given W_i set

$$L(u) = \left(\sum \alpha_i L(u_i) \right) = \sum \alpha_i u_i$$

whenever $u = \sum \alpha_i u_i$

Now check that L is indeed a l.t.

$$\text{s.t. } L(u_i) = W_i \quad \square$$

1. f is onto: if $\forall y \in Y \exists x \in X$
s.t. $f(x) = y$

2. f is 1-1: if $x_1, x_2 \in X$ & $f(x_1) = f(x_2)$
then $x_1 = x_2$

3. We say that f is "invertible" if $\exists g$:

$$g: Y \rightarrow X \quad \text{s.t.} \quad f \circ g = I_Y \quad g \circ f = I_X$$

meaning $\forall y \ f(g(y)) = y \quad \forall x \ g(f(x)) = x$

Claim $f: X \rightarrow Y$ is invertible iff
it is 1-1 and onto.

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we say that two "objects" are isomorphic if there is a bijection between them, which preserves all relevant structure.

Two sets are isomorphic if there is a bijection $f: X \rightarrow Y$

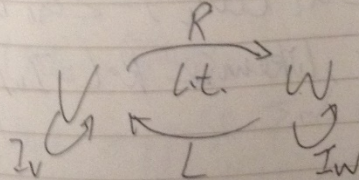
$$\mathbb{Z} \cong \mathbb{Q} \cong \mathbb{R} \cong \mathbb{C}$$

↑
isomorphic.

Def V, W w/ f are "isomorphic" if

$$\exists \text{ lin. } R: V \rightarrow W \text{ \& } L: W \rightarrow V$$

$$\text{s.t. } L \circ R = I_V \quad R \circ L = I_W$$



Thm If $V, W/F$ are fin dim.

then V is isomorphic to W
iff $\dim V = \dim W$

PF. " \Leftarrow " Suppose $\dim V = \dim W = n$

\exists basis $\{u_1, \dots, u_n\}$ of V

basis $\{w_1, \dots, w_n\}$ of W

define $L: W \rightarrow V$, $L(w_i) = u_i$

$R: V \rightarrow W$ by $R(u_i) = w_i$.

Suppose $u \in V$, $u = \sum \alpha_i u_i$

$$L(R(u)) = L(R(\sum \alpha_i u_i))$$

$$= \sum \alpha_i L(R(u_i))$$

$$= \sum \alpha_i L(w_i) = \sum \alpha_i u_i = u$$

$$\Rightarrow L \circ R = I_V, \text{ likewise } R \circ L = I_W$$