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Pf Given $w_i$ set

$$L(u) = \left(\sum \alpha_i L(u_i) =\right) \sum \alpha_i w_i$$

whenever $u = \sum \alpha_i u_i$

Now check that $L$ is indeed a l.t.

s.t. $L(u_i) = w_i$ $\square$

1. $f$ is onto: if $\forall y \in Y \ \exists x \in X$
 s.t. $f(x) = y$

2. $f$ is 1-1: if $x_1, x_2 \in X$ & $f(x_1) = f(x_2)$
 then $x_1 = x_2$

3. We say that $f$ is "invertible" if $\exists g$:
$g: Y \to X$ s.t $f \circ g = I_Y$ $g \circ f = I_X$
 meaning $\forall y \ f(g(y)) = y$ $\forall x \ g(f(x)) = x$

Claim $f: X \to Y$ is invertible iff
 it is 1-1 and onto.

We say that two "objects" are isomorphic if there is a bijection between them, which preserves all relevant structure.

Two sets are isomorphic if there is a bijection $f\ X \to Y$

$$\mathbb{Z} \cong \mathbb{Q} \cong \mathbb{R} \cong \mathbb{C}$$

isomorphic

Def $V, W/F$ are "isomorphic" if

$\exists$ l.t. $R: V \to W$ &

$L: W \to V$

s.t. $L \circ R = I_V \quad R \circ L = I_W$

$$\begin{tikzcd} V \arrow[loop below, "I_V"] \arrow[r, bend left, "R"] \arrow[r, phantom, "\text{l.t.}"] & W \arrow[l, bend left, "L"] \arrow[loop below, "I_W"] \end{tikzcd}$$

Thm If $V, W/F$ are fin dim.
then $V$ is isomorphic to $W$
iff $\dim V = \dim W$

PF. "$\Leftarrow$" Suppose $\dim V = \dim W = n$
$\exists$ basis $\{u_1, \dots, u_n\}$ of $V$
basis $\{w_1, \dots, w_n\}$ of $W$
define $L: W \to V$, by $L(w_i) = u_i$
$R: V \to W$ $R(u_i) = w_i$.
Suppose $u \in V$ $u = \sum \alpha_i u_i$
$$L(R(u)) = L(R(\sum \alpha_i u_i))$$
$$= \sum \alpha_i L(R(u_i))$$
$$= \sum \alpha_i L(w_i) = \sum \alpha_i u_i = u$$
$\Rightarrow L \circ R = I_V$, likewise $R \circ L = I_W$