Mar. 22nd.

Let S^2 be oriended as $\partial D^3 \subset \mathbb{R}^3$ let $\omega \in \Omega^2(\mathbb{R}^3)$ given by $\omega = x\,dy \wedge dz + y\,dz \wedge dx + z\,dx \wedge dy$.

Compute $\int_{S^2} \omega$ (precisely $i: S^2 \to \mathbb{R}^3$.

$i^*: \Omega^2(\mathbb{R}^3) \to \Omega^2(S^2)$.

$\int_{S^2} i^*\omega \rightsquigarrow \int_{S^2} \omega$)

use $\alpha: [0, \infty) \times [0, 2\pi] \times [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\alpha(r, \theta, \phi) = (r\cos\theta\cos\phi, r\sin\theta\cos\phi, r\sin\phi)$

$\det D\alpha = r^2\cos\phi > 0$. So α is orient preser.

$\beta = \alpha|_{r=1}$ $\beta(\theta, \phi) = (\cos\theta\cos\phi, \sin\theta\cos\phi, \sin\phi)$

$$\int_{S^2} \omega = \int_\theta \beta^*\omega = \int \cdots = \int_\theta \cos\phi = 4\pi$$

Def: $\displaystyle \int_M^{\phi} \omega = \sum_{i \in I} \int_M \phi_i \omega$.

Prop: If ϕ_i & ψ_j are P.o.U, then $\displaystyle \int_M^{\phi} \omega = \int_M^{\psi} \omega$

pf: $$\int_M^{\phi} \omega = \sum_{i \in I} \int_M 1\,\phi_i \omega = \sum \int_M (\sum \psi_j)\phi_i \omega$$

$$= \sum_i \sum_j \int \psi_j \phi_i \omega = \sum_j \int (\sum \phi_i)\psi_j \omega$$

$$= \sum \int \psi_j \omega = \int_M^{\psi} \omega$$

properties $\alpha_i \in \mathbb{R}$

1. $\int_M a_1\omega_1 + a_2\omega_2 = a_1\int_M \omega_1 + a_2\int_M \omega_2$

2. $\int_{-M} \omega = -\int_M \omega = \int_M -\omega$.

$-M$ ← with opposite orientation

$$\int_{\partial M} \omega = \int_M d\omega$$