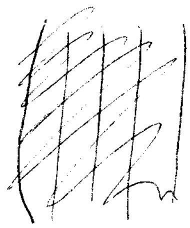
MAT240. Nov. 9, 2006.

Pf Thm 2

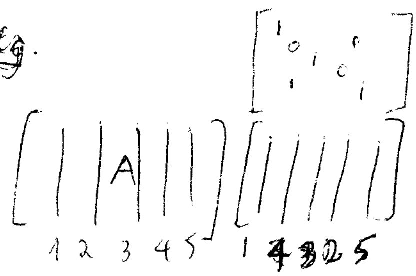
1. $$E^1_{i,j} = \begin{array}{c} \left[\begin{matrix} 1 & & & & \\ & 0 & & 1 & \\ & & 1 & & \\ & 1 & & 0 & \\ & & & & 1 \end{matrix}\right] \end{array} \begin{array}{c} \\ i \\ \\ j \\ \\ \end{array}$$

Claims: $E^1_{i,j} A = \begin{bmatrix} \text{A with rows} \\ \text{i,j swapped} \end{bmatrix}$ $\quad A E^1_{i,j} = \begin{bmatrix} \text{A with columns} \\ \text{i \& j swapped} \end{bmatrix}$

$$\left(E^1_{i,j}\right)^{-1} = E^1_{i,j}.$$



eg.



$$E^1_{i,j} \cdot E^1_{i,j} = I. \checkmark$$

2. $$E^2_{i,c} = \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & c & \\ & & & 1 \end{bmatrix} \begin{array}{c} \\ \\ \\ i \\ \\ \end{array}$$

Claims: a) $E^2_{i,c} A = \begin{bmatrix} \text{A with row i} \\ \text{multiplied by c} \end{bmatrix}$ $\quad$ b) $A E^2_{i,c} = \dots$

c) $\left(E^2_{i,c}\right)^{-1} = E^2_{i,c^{-1}}$

3. $$E^3_{i,j,c} = \begin{array}{c} \begin{array}{ccccc} & & & j & \end{array} \\ \left[\begin{matrix} 1 & & & c & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{matrix}\right] \end{array} \begin{array}{c} i \\ \\ \\ \\ \\ \end{array}$$

Claims: a) $E^3_{i,j,c} A = \begin{bmatrix} \text{A with row j} \\ \text{times c added} \\ \text{to row i} \end{bmatrix}$ $\quad$ b)

c) $\left(E^3_{i,j,c}\right)^{-1} = E^3_{i,j,-c}$

$a := a+b$
$b := a-b$
$a := a-b$.

instead of using var c.

$$\left(\left(\begin{array}{l} c := a \\ a := b \\ b := c \end{array}\right)\right.$$

i.e. don't need first operation.

Claim: If $P: V \to V$ is invertible & H is a subspace of V then $P(H) = \{P(h) ; h \in H\}$.
is a subspace of V and $\dim P(H) = \dim H$.

Pf. If h_i is a basis of H, then $P(h_i)$ is a basis of $P(H)$ with the same # of elts.