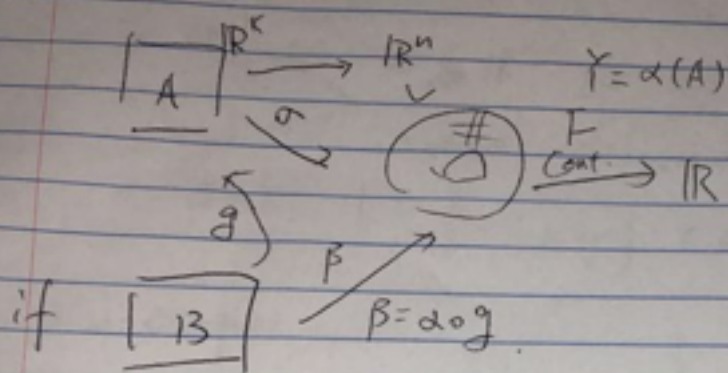


Jan. 19th

Def: $\int_Y f \, dV = \int_A (f \circ \alpha) V(D\alpha) \stackrel{\text{Thm}}{=} \int_B (f \circ \beta) V(D\beta)$



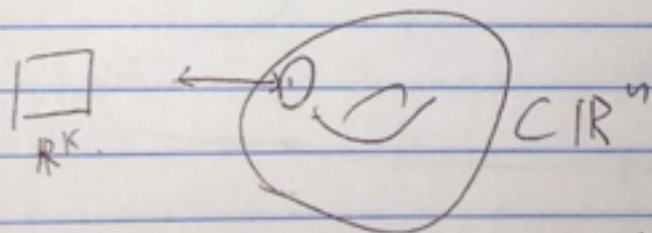
$$\int_M \omega = \int_{\alpha(M)} \omega$$

M : "A nice and smooth k -dimensional subset of \mathbb{R}^n "

k -Klein $\subset \mathbb{R}^4$

$$O(n) = \{A \in M_{n \times n} : A^T A = I\} \subset \mathbb{R}^{n^2}$$

$$O(2) = S^1 \cup S^1 \subset \mathbb{R}^4$$



Every point has a neighborhood looks like that.

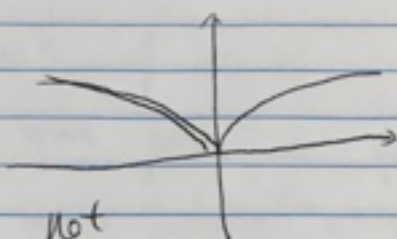
Def: A k -dim manifold (w/o bndry, of class C^r , $r \geq 1$) in \mathbb{R}^n , is a subset $M \subset \mathbb{R}^n$ s.t. each $p \in M$ has an open nbhd V s.t.

\exists open $U \subseteq \mathbb{R}^k$ & a C^r homeomorphism $\alpha: U \rightarrow V$, whose differential $D_x \alpha$ has rank k for every $x \in U$.

Non-examples:

① $\alpha: \mathbb{R} \rightarrow \mathbb{R}^2$

$t \mapsto (t^3, t^2)$



$D_x \alpha = (3t^2, 2t)$ has rank 1 $\textcircled{\neq}$ $t=0$

α is a homeomorphism.

②

