MAT 240 - tutorial

Nov. 23, 2c

2.4

9. need to also find left inverse

~~M_{AB}~~

$L_A : F^n \to F^n$

$x \mapsto Ax$

$L_{AB} = L_A \circ L_B$

$L_{AB}(x) = ABx = AL_B(x) = L_A(L_B(x))$

AB invertible

$\Rightarrow L_{AB}$ "

$\Rightarrow L_{AB}$ is 1-1

$L_B(x) = 0$

$L_{AB}(x) = 0$

$\Rightarrow L_B$ is 1-1

$\Rightarrow$ B invertible

$\therefore$ A invertible

.2

21. If $A = (I_m \; 0)$ $\quad B = \begin{bmatrix} I_m \\ 0 \end{bmatrix}$

$\exists P, Q$ invertible s.t.

$PAQ = (I_m \; 0)$

$PAQ \begin{pmatrix} I_m \\ 0 \end{pmatrix} = I_m$ can also use this method for 2.4 Q.9

$AQ \begin{pmatrix} I_m \\ 0 \end{pmatrix} = P^{-1}$

$\underbrace{AQ \begin{pmatrix} I_m \\ 0 \end{pmatrix} P}_{} = I_m$

$\boxed{3.3}$

10. $Ax=b \iff A$ is onto.
Since rank $= m \Rightarrow$ onto.

$A^{n\times n}$

$\boxed{4.1 \, \#3}$ $\det(KA) = K^n \det A$

$\det(Kv_1 \cdots Kv_n)$
$= K\det(v_1, Kv_2, \dots, Kv_n)$
$= K^n \det(v_1, \dots, v_n)$

$\boxed{4.3}$. 10. $\det(M^K) = 0$. Since it's rank preserving, rank M must be 0, which is not invertible
$\quad\; \parallel$
$\det(M)^K$

11. $\det(M^t) = \det(-M)$
$\quad \parallel \qquad = (-1)^n \det(M)$
$\det(M) \quad = -\det(M)$
~~contradiction~~.
$\Rightarrow 2\det M = 0$, $2 \neq 0$
$\Rightarrow M$ not invertible

22. a) $M = \left(T(1) \middle| T(x) \middle| \cdots \middle| T(x^n) \right)$

$T(x^k) = (c_0^k, \dots, c_n^k)$
$= \sum_{i=0}^{n} c_i^k e_i$

$$= \begin{pmatrix} 1 & c_0 & & c_0^n \\ \vdots & \vdots & \cdots & \vdots \\ 1 & c_n & & c_n^n \end{pmatrix}$$

b) $\det(M) \neq 0$. b/c Langrange polynomials are lin indep, & maps from basis to basis so invertible. so $\neq 0$

↳ over.

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Lag. polynomial. $f_i(c_j) = \delta_{ij}$

$\{f_0, \dots, f_n\}$ is lin indep.

$\Rightarrow$ basis in $P_n(F)$

$T(f_i) = e_i$

$\Rightarrow$ T is iso.

? $\Rightarrow \det(M) \neq 0$.

c) eg. n=1:

$$\det\begin{pmatrix} 1 & c_0 \\ 1 & c_1 \end{pmatrix} = c_1 - c_0.$$

n=2:

$$\det\begin{pmatrix} 1 & c_0 & c_0^2 \\ 1 & c_1 & c_1^2 \\ 1 & c_2 & c_2^2 \end{pmatrix} = (c_2 - c_0)(c_2 - c_1)(c_1 - c_0)$$

$$= \det\begin{pmatrix} 1 & c_0 & 0 \\ 1 & c_1 & c_1^2 - c_1 c_0 \\ 1 & c_2 & c_2^2 - c_2 c_0 \end{pmatrix}$$

$$= \det\begin{pmatrix} 1 & 0 & 0 \\ 1 & c_1 - c_0 & c_1(c_1 - c_0) \\ 1 & c_2 - c_0 & c_2(c_2 - c_0) \end{pmatrix}$$

$$= \det\begin{pmatrix} c_1 - c_0 & c_1(c_1 - c_0) \\ c_2 - c_0 & c_2(c_2 - c_0) \end{pmatrix}.$$

$$= (c_1 - c_0)(c_2 - c_0)\det\begin{pmatrix} 1 & c_1 \\ 1 & c_2 \end{pmatrix} \quad ''$$

$$\det M = \begin{pmatrix} 1 & c_0 & c_0^{n-1} & 0 \\ \vdots & \vdots & \vdots & c_1^n - c_1^{n-1} c_0 \\ 1 & c_n & c_n^{n-1} & \vdots \\ & & & c_n^n - c_n^{n-1} c_0 \end{pmatrix}.$$

$\vdots$

28. a) $\pi(K_{\eta}) =$

b) $\supseteq$ $\because$ $T(y_i) = 0$ $\because$ we have 2 columns equal.

$\subseteq$

$$T(y) = 0 \Rightarrow \det \begin{pmatrix} y & y_1 & \cdots & y_n \\ y' & \vdots & & \\ \vdots & \vdots & & \\ y^{(n)} & y_1^{(n)} & \cdots & y_n^{(n)} \end{pmatrix} = 0.$$

$$\begin{matrix} y(t) \\ \vdots \\ y^{(n)}(t) \end{matrix} \quad \sum a_i \begin{pmatrix} y_i(t) \\ y_i'(t) \\ \vdots \\ y_i^{(n)}(t) \end{pmatrix} = 0. \quad \text{for some } (a_0, \ldots, a_n) \neq 0$$

$$a_0 y(t) + \sum a_i y_i(t) = 0.$$

$a_0 \neq 0$ $\because$ $\{y_1, \ldots, y_n\}$ are lin indep.

$\exists y \in \text{span}\{y_1, \ldots, y_n\}$.