

$$f(x) - \varepsilon$$

claim 2: if $y \in V$, then $a < f(y) \leq b < f(x) + \varepsilon$



Nov 22 Riddle Along = $\mathcal{G}(x \underset{\in \mathbb{R}^n}{\sim} *) = \text{Spiral} \approx \text{Topo } \approx \text{S}^2$

$$\mathcal{G}(* \underset{\in \mathbb{R}^n}{\sim} *) = .$$

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Read along = 38 (~), 35.

Challenge: Show a metric \Rightarrow embeds in cube by passing Urysohn

today: $T_{3.5} \hookrightarrow$ subset of a cube I^n

Prop: X is $T_{3.5}$ if

$\{[f \neq 0] \mid f: X \rightarrow \mathbb{R} \text{ cont}\}$ is a basis for the topology of X

Pf:

\Rightarrow let $U \subset X$ be open & let $x \in U$.

by $T_{3.5}$, can find $f: X \rightarrow \mathbb{R}$ s.t.

$f(x) = 1 \quad f(U) = \{0\} \quad \text{let } B = \{[f \neq 0] \mid f: X \rightarrow \mathbb{R} \text{ cont}\}$

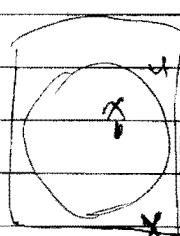
$\therefore [f \neq 0] \in B$ and $x \in [f \neq 0] \subset U$. \checkmark

Notation:

$$[f \neq 0] = \{x : f(x) \neq 0\}$$

$E = f^{-1}(R \setminus \{0\})$ is open

$$[f > 0] = \{x : f(x) > 0\}$$



\Leftarrow Suppose $A \subset X$ is closed & $x \notin A$

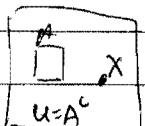
by the fact that B is a basis, can find f s.t. $x \in [f \neq 0] \subset A^c$

$$\text{So } f(A) = 0 \quad f(x) = c \neq 0$$

$$\text{finally let } \tilde{f}(x) = \frac{f(x)}{c} \Rightarrow \tilde{f}(A) = 0 \quad \tilde{f}(x) = 1$$

in order to get the function bdd. should let $\tilde{f}(x) = \min[\max(\frac{f(x)}{c}, 0), 1]$

and still have $\tilde{f}(A) = 0 \cdot \tilde{f}(x) = 1$



X is $T_{3.5} \Leftrightarrow X$ is a subset of I^A

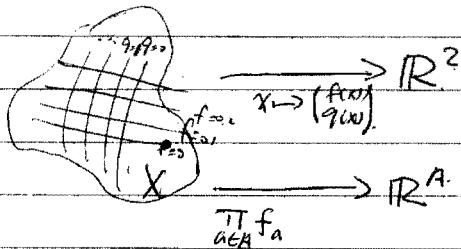
meaning: f "embeds" in I^A

meaning: \exists cont 1-1 $\phi: X \rightarrow I^A$

s.t. $\phi: X \rightarrow \phi(X)$ is a homeomorphism

a cont function which is invertible

i.e. $\phi(u)$ is open $\phi(x)$ when U is open in X



Given X , let $C_x = C(X, I) = \{f | f: X \rightarrow I \text{ cont}\}$

$\phi: X \rightarrow I^{C_x}$ by $\phi = \prod_{f \in C_x} f \Leftrightarrow \phi_f = f$

$$\Leftrightarrow \phi(x)_f = f(x)$$

Thm: ϕ is an embedding iff X is $T_{3.5}$.

PF: ϕ is 1-1: If $x \neq y \in X$ use $T_{3.5}$ to find

$f: X \rightarrow I$ s.t. $f(x) = 0, f(y) = 1$

then, $f(x) = \phi(x)_f \neq \phi(y)_f = f(y)$

So $\phi(x) \neq \phi(y)$.

now let U be a basic open set

" $\{f \neq 0\}$ "

then, $\phi(U) = \phi(\{f \neq 0\}) = \phi(x) \cap \prod_f (\{f \neq 0\})$

$\subset U$

$\supset U$

So $\phi(U)$ is open in $\phi(X)$

\Rightarrow easy part. [Exercise]