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number: 1001081517　　Homework Assignment2.
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P3.　　$6+4+6=16$

3(a) Solution: Let $A=(2,-5,-1)$, $B=(0,4,6)$, $C=(-3,7,1)$
The vector beginning at A and terminating at B.
has coordinates $A-B=(2,-5,-1)-(0,4,6)=(2,-9,-7)$
The vector $\vec{AC}$ has coordinates
$A-C=(2,-5,-1)-(-3,7,1)=(5,-12,-2)$
Hence the equation of plane is:
$x=(2,-5,-1)+s(2,-9,-7)+t(5,-12,-2)$, $s,t\in\mathbb{R}$
(x denotes an arbitrary point in the plane)

P12.
1. (a) True
By VS4. $\forall x\in V$, $\exists y\in V$ s.t $x+y=0$.
$\because +: V\times V\to V$
$\therefore 0\in V$
(b) False.
If there exists a $0'$, $0'$ has the property of 0.
Then By VS3 $a+0'=a$ $a+0=a \Rightarrow a+0'=a+0$
By cancellation law, $0=0'$, Hence there's only one 0.
(c) False
When $x=0$, then $a\cdot 0=0$, $b\cdot 0=0 \Rightarrow ax=bx$
but a is an arbitrary element and b is an arbitrary element in F

d) False

If $a=0$, then $ax=0$, $ay=0 \Rightarrow ax=ay$

but, x, y can be an arbitrary element in V

e) True

$\because F^n = \{ \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \mid x_i \in F \}$

$M_{n\times1}(F) = \{ \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \mid x_i \in F \}$

$\therefore F^n = M_{n\times1}(F)$

f) False

m rows n columns.

g) False

$P(x) = x^2+1 \quad Q(x) = x+1 \quad \deg P = 2 \quad \deg = 1.$

$P(x)+Q(x) = x^2+x+2$

6/6

h) False

$f(x) = x^2+1 \quad g(x) = -x^2+1 \quad \deg f = \deg g = 2$

$f(x)+g(x) = 2 \quad \deg = 0$

i) True

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \quad a_n \neq 0, \deg = n$

$cf(x) = ca_n x^n + ca_{n-1}x^{n-1} + \cdots + ca_1 x + ca_0 \quad c, a_n \neq 0, \deg = n$

j) True

$f(x) = a \quad a \in F \wedge a \neq 0. \Rightarrow \deg f = 0.$

k) True

$f = g \Leftrightarrow f(s) = g(s)$ for each $s \in S$

Student name: Wu Boyang MA 24... Assignment 3.
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P14
7. Proof: ① We need to show $f=g$, we first get the solutions of $f=g$.

$\Rightarrow f=g \Rightarrow 2t+1 = 1+4t-2t^2$

$\Rightarrow t(t-1)=0$

$\Rightarrow t=0,1 \Rightarrow$ for all $t\in S$, $f=g$.

② We need to show $f+g=h$, $(f+g)(t)=f(t)+g(t)$

$\Rightarrow (f+g)(0) = 1+1 = 2$

$(f+g)(1) = 2+1+1+4-2 = 6$.

$h(0) = 1+1 = 2 = (f+g)(0)$

$h(1) = 5+1 = 6 = (f+g)(1)$

$\Rightarrow$ for all $t\in S$ $f+g=h$.

Hence $f=g$, $f+g=h$

P15
18. Solution:

V is not a vector space over $\mathbb{R}$

Now we will prove that.

If V is a vector space, it needs to satisfy VS1 (the commutativity of addition) For all $x,y\in V$, $x+y=y+x$

Then $(a_1,a_2)+(b_1,b_2)=(a_1+2b_1, a_2+3b_2)$

$(b_1,b_2)+(a_1,a_2)=(b_1+2a_1, b_2+3a_2)$

Hence $(a_1,a_2)+(b_1,b_2)\neq(b_1,b_2)+(a_1,a_2)$ ↯

So V is not a vector space over $\mathbb{R}$.

example? 4/6