28.09.06 Tutorial

Solving equations

1) Solve the system of linear equations (on p. 33)

solution in $\mathbb{R}$

① $3x_1 - 7x_2 + 4x_3 = 10$

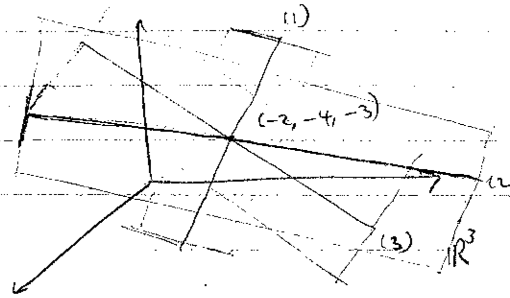
② $x_1 - 2x_2 + x_3 = 3$

③ $2x_1 - x_2 - 2x_3 = 6$

To solve this system $\Leftrightarrow$ find all the numbers (x_1, x_2, x_3) which satisfy this system we may add equatio[ns] in the system and multiply equations by numbers ($\neq 0$)

we would like to get rid of x_1 in the second & the third eq'ns. Multiply (2) by 3 and substract it from the (1)

$$\begin{cases} 3x_1 - 7x_2 + 4x_3 = 10 & 1 \\ \quad -x_2 + x_3 = 1 & 2 \\ 2x_1 - x_2 - 2x_3 = 6 & 3 \end{cases} \sim (1)\cdot 2 - (3)\cdot 3 \Rightarrow$$



$$\Rightarrow \begin{cases} 3x_1 - 7x_2 + 4x_3 = 10 \\ \quad -x_2 + x_3 = 1 \\ \quad -11x_2 + 14x_3 = 2 \end{cases} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \underset{\sim}{(2)\cdot 11} - (3) \quad \begin{cases} 3x_1 - 7x_2 + 4x_3 = 10 \\ \quad -x_2 + x_3 = 1 \\ \quad -3x_3 = 9 \Rightarrow x_3 = -3 \end{cases}$$

plug into (2) $x_2 = -4$ then plug into (1)

$3x_1 - 7(-4) + 4(-3) = 10 \qquad x_1 = -2$

<u>Conclusion</u>: $\exists!$ (there is a unique) sol'n of this system $(-2, -4, -3) \in \mathbb{R}^3$

2) $\begin{cases} 2x_1 - 2x_2 - 3x_3 = -2 & (1) \\ 3x_1 - 3x_2 - 2x_3 + 5x_4 = 7 & (2) \\ x_1 - x_2 - 2x_3 - x_4 = -3 & (3) \end{cases} \xrightarrow{(2)-(3)3} \begin{cases} 2x_1 - 2x_2 - 3x_3 & (1) \\ 3x_1 - 3x_2 - 2x_3 + 5x_4 & (2) \\ 4x_3 + 8x_4 & (3) \end{cases}$

$\xrightarrow{(1)3-(2)\cdot 2} \begin{cases} 2x_1 - 2x_2 - 3x_3 = -2 & (1) \\ -5x_3 - 10x_4 = 20 & (2) \\ 4x_3 + 8x_4 = 16 & (3) \end{cases} \xrightarrow{(2)4+(3)\cdot 5} \begin{cases} 2x_1 - 2x_2 - 3x_3 = -2 & (1) \\ -5x_3 - 10x_4 = -20 & (2) \\ 0 = 0 & (3) \end{cases}$

(2) $\Rightarrow -5x_3 = -20 + 10x_4 \Leftrightarrow x_3 = 4 - 2x_4$

plug this into (1)

(1) $\Leftrightarrow 2x_1 - 2x_2 - 12 + 6x_4 = -2$

$\Leftrightarrow x_1 = x_2 + 6 - 3x_4 - 1$

We get $x_1 = x_2 - 3x_4 + 5$ | Now We treat x_2 and x_4 as the parameters

⊛ $x_2 = 4 - 2x_4$

Conclusion: the set of solutions of the given system depends on two parameters

For example: put $x_2 = x_4 = 0$. Then from ⊛ $x_1 = 5$, $x_3 = 4$, so $(5, 0, 4, 0)$

is a solution.

3 (a) 3) Is it possible to express V_1 as a linear const. of V_2, V_3

See 1.4

$V_1 = (-2,0,3)$, $V_2 = (1,3,0)$, $V_3 = (2,4,-1)$

we are looking for the constants a, b s.t. (*) $V_1 = aV_2 + bV_3$

$$* \Leftrightarrow \begin{cases} a+2b = -2 \\ 3a+4b = 0 \\ -b = 3 \end{cases} \sim \begin{array}{l} b=-3 \\ a=4 \end{array}$$

Not correct ! need to have another constan

Now, we are looking for the constants a, b, c s.t. $c \neq 0$,

(*) $cV_1 = aV_2 + bV_3$ then it would follow that $V_1 = \frac{a}{c} V_2 + \frac{b}{c} V_3$

$$(*) \Leftrightarrow \begin{cases} a+2b+2c = 0 \\ 3a+4b = 0 \\ -b-3c = 0 \end{cases} \overset{(1)3-(2)}{\sim} \begin{cases} a+2b+2c = 0 \\ 2b+6c = 0 \\ b+3c = 0 \end{cases}$$

$$\overset{(2)-(3)2}{\sim} \begin{cases} a+2b+2c = 0 \\ b+3c = 0 \\ 0 = 0 \end{cases} \sim \begin{cases} a+2b+2c = 0 & (1) \\ b+3c = 0 & (2) \end{cases}$$

(2) $\Rightarrow b = -3c$

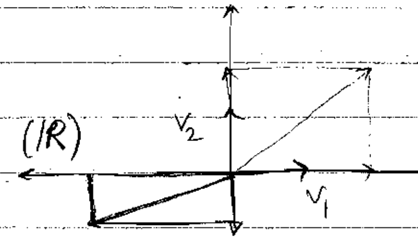
Plug in (1)

(1) $\Leftrightarrow a - 6c + 2c = 0 \Leftrightarrow a = 4c$

Then $(4c, -3c, c)$ is the solution of this systems

4) Show that

$$\operatorname{span}\left\{\begin{pmatrix}1&0\\0&0\end{pmatrix},\begin{pmatrix}0&1\\0&0\end{pmatrix},\begin{pmatrix}0&0\\1&0\end{pmatrix},\begin{pmatrix}0&0\\0&1\end{pmatrix}\right\} = M_{2\times2}(\mathbb{R})$$



Meaning: take any $\begin{pmatrix}x_1&x_2\\x_3&x_4\end{pmatrix}\in M_{2\times2}(\mathbb{R})$

Then I'm able to find $a,b,c,d\in\mathbb{R}$

$$\begin{pmatrix}x_1&x_2\\x_3&x_4\end{pmatrix} = a\begin{pmatrix}1&0\\0&0\end{pmatrix}+b\begin{pmatrix}0&1\\0&0\end{pmatrix}+c\begin{pmatrix}0&0\\1&0\end{pmatrix}+d\begin{pmatrix}0&0\\0&1\end{pmatrix}$$

or $$\begin{pmatrix}x_1&x_2\\x_3&x_4\end{pmatrix} = \begin{pmatrix}a&b\\c&d\end{pmatrix}$$

<u>So</u>, take $a=x_1$, $b=x_2$, $c=x_3$, $d=x_4$

$$\begin{pmatrix}x_1&x_2\\x_3&x_4\end{pmatrix} = x_1\begin{pmatrix}1&0\\0&0\end{pmatrix}+x_2\begin{pmatrix}0&1\\0&0\end{pmatrix}+x_3\begin{pmatrix}0&0\\1&0\end{pmatrix}+x_4\begin{pmatrix}0&0\\0&1\end{pmatrix}$$

* Goal for the next few weeks

All vectorspaces are the "same" as F^n? $F^n = \left\{\begin{pmatrix}a_1\\ \vdots\\ a_n\end{pmatrix}\right\}$ all vector spaces the same

but in what sense?

1. What's "same"
2. So what? $\{1, \ldots 73\}$ good to number elements (easy to identify students with number)
3. So why bother with vector spaces?
4. How is it prove?

$$\{\text{cow}, \text{dog}, \text{house}\}$$
$$\updownarrow \quad \updownarrow \quad \updownarrow$$
$$\{1, 2, 3\}$$

$\{1, 2, 7, 13\}$ with glasses

$\{23, 12, 4, 5\}$ without glasses