Make up class 30th Thur. Mar. Hour 069

Read along: 37-38

Stoke's : $w \in \Omega^{k-1}$ (M^k compact / orientable) $\Rightarrow \int_M dw = \int_{\partial M} w$

Integration : $\int_M w$

$\Omega^0(\mathbb{R}^3)$ | $\Omega^1(\mathbb{R}^3)$ | $\Omega^2(\mathbb{R}^3)$

$\sum s_i F(p_i)$ | $\int a\cdot\dot{r} = \int a\cdot\vec{T}\,dv$ | $\int_S b\cdot\vec{n}\,dv$

[1] [2]

curve 0 → 1 : $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \leadsto w = \sum a_i dx_i$, $r_i = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix}$

$$\int_\Gamma w = \int_{[0,1]} r^* w = \int_{[0,1]} \sum a_i dr_i = \int_{[0,1]_t} \sum a_i \dot{r}_i\, dt$$

(↑ evaluated at r_i)

$$= \int_{[0,1]} a \cdot \dot{r} \overset{*}{=} \int_{[0,1]} a\cdot\vec{T}\cdot\|\dot{r}\| = \int_{[0,1]} a\cdot\vec{T}\,dv$$

(vector)

*: if we write $\dot{r} = \|\dot{r}\|\cdot\vec{T}$ ↪ the unit tangent to Γ

k=1

[1] : curve Γ, function F

$$\int_\Gamma (\text{grad} F)\cdot\vec{T}\,dv = F(r(1)) - F(r(0))$$

↓ increase ↓ direction to whole curve

Integral of "total climb rate"

surface
1→2 : $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \rightsquigarrow w = b_1 dx_{2,3} + cp, \quad \sigma = \begin{pmatrix} \sigma_1(x,y) \\ \sigma_2(x,y) \\ \sigma^3(x,y) \end{pmatrix}$

$$\int_S w = \int_{D^2_{x,y}} \sigma^* w = \int_{D^2} b \cdot \vec{n}\, v\left(\frac{\partial \sigma}{\partial x}, \frac{\partial \sigma}{\partial y}\right) = \int_S b \cdot \vec{n}\, dv \quad \#$$

$$\sigma^* w = \sigma^*(b_1 dx_2 \wedge dx_3 + c.p.)$$
$$= b_1\, d\sigma_2 \wedge d\sigma_3 + cp = b_1\left(\frac{\partial \sigma_2}{\partial x} dx + \frac{\partial \sigma_2}{\partial y} dy\right) \wedge \left(\frac{\partial \sigma_3}{\partial x} dx + \frac{\partial \sigma_3}{\partial y} dy\right) + c.p.$$

~~$*: \partial\sigma_1$~~ ~~$= \left[b_1\left(\frac{\partial \sigma_2}{\partial x}\frac{\partial \sigma_3}{\partial y} - \frac{\partial \sigma_3}{\partial x}\frac{\partial \sigma_2}{\partial y}\right) + c.p.\right]$~~ $dx \wedge dy$

$$= b_1\left(\frac{\partial \sigma_2}{\partial x} \cdot \frac{\partial \sigma_3}{\partial y} - \frac{\partial \sigma_3}{\partial x}\frac{\partial \sigma_2}{\partial y}\right) dx \wedge dy + c.p.$$

$$= \left[b_1 \left(\frac{\partial \sigma}{\partial x} \times \frac{\partial \sigma}{\partial y}\right)_1^{*} + c.p.\right] dx \wedge dy$$

$$= \sum b_i \left(\frac{\partial \sigma}{\partial x} \times \frac{\partial \sigma}{\partial y}\right)_i dx \wedge dy$$

$$*: \begin{pmatrix} \frac{\partial \sigma_1}{\partial x} \\ \frac{\partial \sigma_2}{\partial x} \\ \frac{\partial \sigma_3}{\partial x} \end{pmatrix} \times \begin{pmatrix} \frac{\partial \sigma_1}{\partial y} \\ \frac{\partial \sigma_2}{\partial y} \\ \frac{\partial \sigma_3}{\partial y} \end{pmatrix}$$

$$= b \cdot \left(\frac{\partial \sigma}{\partial x} \times \frac{\partial \sigma}{\partial y}\right) dx \wedge dy$$

$$= b \cdot \vec{n} \cdot v\left(\frac{\partial \sigma}{\partial x}, \frac{\partial \sigma}{\partial y}\right) dx \wedge dy$$

↑ unit normal to s

where $v_1 \times v_2 = v(v_1, v_2) \cdot \vec{n}$, the unit normal to v_1 and v_2

$\frac{\partial \sigma}{\partial x}, \frac{\partial \sigma}{\partial y}$ push forwards of the std e_1, e_2 using σ

~~Visa Visualize~~

Visualize #? flow in $\mathbb{R}^3$, surface in $\mathbb{R}^3$

Surface has normal at any point, ~~at~~ and take normal.

take ~~at~~ component at the direction of normal.

v.f. $\parallel S \Rightarrow 0$, v.f. $\perp S \Rightarrow$ the size of the v.f.

meaning how much the flow "flowes" through the surface.

So it measures the flow of b through S.

k=2

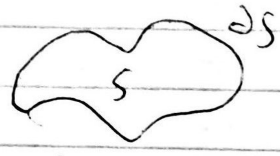
[2] : $\int_S (\mathrm{Curl}\, a) \cdot \vec{n}\, dv = \int_S dw = \int_{\partial S} w = \int_{\partial S} a \cdot \vec{T} dv.$

Surfaces S

v.f. a

↑

since we use a in $\Omega^1(\mathbb{R}^3)$



flow in $\mathbb{R}^3$, = a

Curl.a measures # flow tends to spin

Eeah spin measures only the part of spinning, whose access is $\perp$ to the surface.

5:55

lhs : at around ~~5:55~~. WATCH THE VIDEO

local spinning in the plane of s

rhs : easier. flows parallel to the boundry, circulation of a. along ∂S