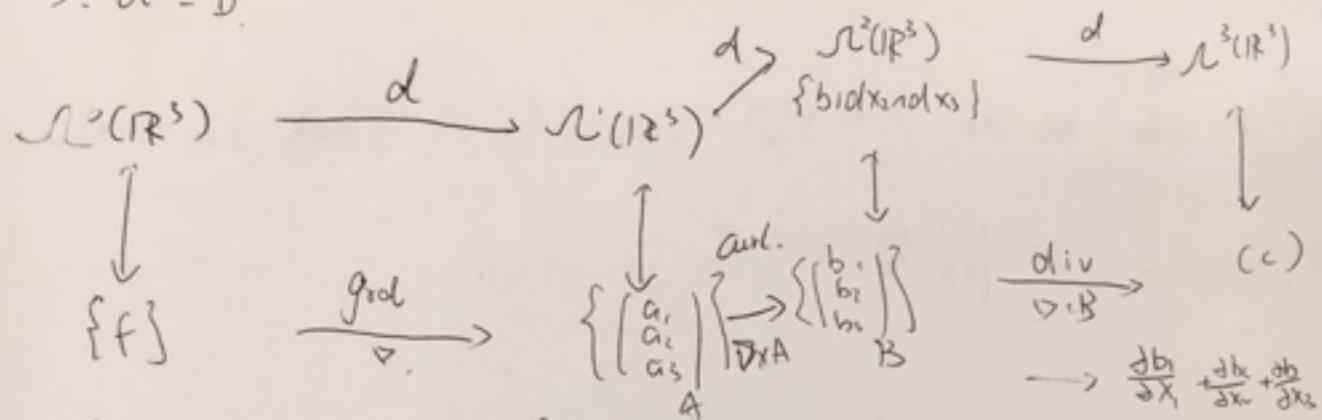


Thm:  $\exists$  linear  $d: \mathcal{L}^k \rightarrow \mathcal{L}^{k+1}$ , s.t.

$$1. f \in \mathcal{L}^0 \Rightarrow (df)(\{\}) = D_{\{\}} f = \sum \frac{\partial f}{\partial x_i} dx_i$$

$$2. d(\omega \wedge \eta) = (\omega \wedge) \eta + (-1)^{kl} \omega \wedge d\eta$$

$$3. d^2 = 0$$



If  $f: \mathbb{R}^n \rightarrow \mathbb{R}$   $\text{grad } f$  = direction in which  $f$  increases the fastest.

Pf:  $D_{(x,v)} f = \sum \frac{\partial f}{\partial x_i}(x) \cdot v_i = \langle \text{grad } f, v \rangle$  — this quantity is maximal when  $v$  points in direction of  $\text{grad } f$ .

Pf: 1-3. imply  $d(\sum a_i dx_i) = \sum \frac{\partial a_i}{\partial x_j} dx_i \wedge dx_j$ . proves uniqueness.

$\boxed{B} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

$\begin{array}{ccc} \nearrow & \nearrow & \nearrow \\ \frac{\partial b_1}{\partial x_1} > 0 & & \\ \downarrow & \downarrow & \downarrow \\ x_1 & & \end{array}$  something is added to the system.

$\begin{array}{ccc} & & \nearrow \\ & & \frac{\partial b_1}{\partial x_1} < 0 \\ \downarrow & \downarrow & \downarrow \\ & & \end{array}$   $\frac{\partial b_1}{\partial x_1} + \frac{\partial b_2}{\partial x_2} + \frac{\partial b_3}{\partial x_3}$

$\mathcal{L}^1 \rightarrow \mathcal{L}^2$

$$d(a_1 dx_1 + a_2 dx_2 + a_3 dx_3)$$

$$= \begin{pmatrix} \frac{\partial a_3}{\partial x_2} - \frac{\partial a_2}{\partial x_3} \\ \frac{\partial a_1}{\partial x_3} - \frac{\partial a_3}{\partial x_1} \\ \frac{\partial a_2}{\partial x_1} - \frac{\partial a_1}{\partial x_2} \end{pmatrix} \begin{pmatrix} dx_2 \wedge dx_3 \\ dx_3 \wedge dx_1 \\ dx_1 \wedge dx_2 \end{pmatrix} \cdot \nabla \times f$$

curl ~~not~~ measures rotation of a body inner side in a flow.

measures creation/annihilation of flux.

$$\overrightarrow{\omega} \quad \uparrow \square \quad \uparrow \frac{\partial \alpha_2}{\partial x_1}$$

$$-\frac{\alpha_1}{\partial x_2}$$

Claim: If ~~form~~ for  $\omega = \sum \alpha_i dx_i \in \mathcal{N}^k(\mathbb{R}^n)$ , we set  $d\omega = \sum \frac{\partial \alpha_i}{\partial x_j} dx_j \wedge dx_i$

$$\sum dx_j \wedge \frac{\partial \alpha_i}{\partial x_j} dx_i = \sum dx_j \wedge \frac{\partial \omega}{\partial x_j} = d\omega.$$