2. $\dim\{0\} = 0$.

$\beta = \phi$ ✓ $\cancel{\{0\}}$ $|\beta| = 0$.

3. $\dim F = 1$ $\beta = (1)$.

4. $\mathbb{C}$ is a V.S over $\mathbb{R}$.

$\dim_{\mathbb{R}} \mathbb{C} = 2$. $\beta = (1, i)$

$\mathbb{C} \ni z = a \cdot 1 + b \cdot i$ so

$\mathrm{span}(\beta) = \mathbb{C}$,

if $a \cdot 1 + b \cdot i = 0 \Rightarrow a = b = 0$.

So β is lin. indep.

5. $\dim M_{m\times n}(F) = mn$

basis $\left\{ \begin{array}{c} \\ i \\ \\ \end{array} \begin{pmatrix} 0 & \cdots & \cdots & 0 \\ \vdots & & & \vdots \\ & & 1 & \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & 0 \end{pmatrix} \right\}_{j} = \beta$ $|\beta| = mn$

$\dim \left\{ \begin{pmatrix} \vdots & \vdots \\ \vdots & \vdots \end{pmatrix} \right\} = 4$

6. $\dim P_n(F)$

$\cup$

$\{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0\}$

$\beta = \{x^n, x^{n-1}, \cdots, x, 1\}$

$|\beta| = n+1$

7. $\dim P(F) = \infty$ does not make sense.

basis $\beta = (1, x, x^2, \cdots\cdots)$

$|\beta| = \infty$

Lemma ("The replacement lemma")

Suppose $G, L \subset V$

1. G generates $\operatorname{span}(G) = V$
2. L is lin indep & $|L| = n$

Then there exist $R \subseteq G$, $|R| = n$.

s.t. $(G \setminus R) \cup L$