

Pf of property 2.

If the row in question is the top row, observe

$$\begin{pmatrix} ar^1 + br'' \\ \hline \hline \hline \hline \end{pmatrix} = a \begin{pmatrix} r^1 \\ \hline \hline \hline \hline \end{pmatrix} + b \begin{pmatrix} r'' \\ \hline \hline \hline \hline \end{pmatrix}$$

$A$ 
 $A'$ 
 $A''$

$$A_{ij} = \begin{cases} A'_{ij} = A''_{ij} & i > 1 \\ aA'_{ij} + bA''_{ij} & i = 1 \end{cases}$$

$$\begin{aligned} |A| &= \sum (-1)^{i+j} A_{ij} |\tilde{A}_{ij}| \\ &= \sum (-1)^{i+j} (aA'_{ij} + bA''_{ij}) |\tilde{A}_{ij}| \\ &= a \sum (-1)^{i+j} A'_{ij} |\tilde{A}_{ij}| + b \sum (-1)^{i+j} A''_{ij} |\tilde{A}_{ij}| \\ &= a|A'| + b|A''| \end{aligned}$$

otherwise,  $\det A$  is a linear combination of  $\det \tilde{A}_{ij}$  and in each  $\tilde{A}_{ij}$  the row in question is bumped up once.

