Def: Given $A \in M_{m \times n}$, $B \in M_{n \times p}$

Define a new matrix: $C = A \cdot B \in M_{m \times p}$

$$C_{ki} = \sum_{j=1}^{n} a_{kj} b_{ji}$$

THEOREM: In our situation

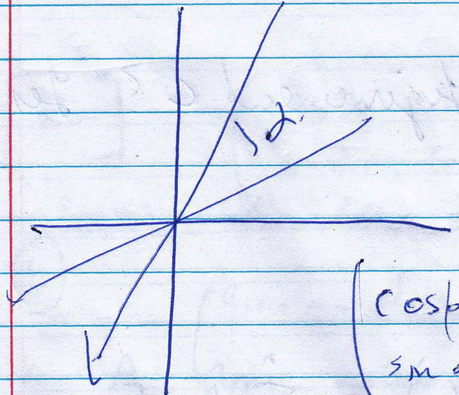
$$\underset{m \times p}{[T \circ S]_\alpha^\gamma} = \underset{m \times n}{[T]_\beta^\gamma} \cdot \underset{n \times p}{[S]_\alpha^\beta}$$

Example 1

$$\underset{2 \times 3}{\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}} \underset{3 \times 2}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{pmatrix}} = \underset{2 \times 2}{\begin{pmatrix} -2 & 5 \\ -2 & 11 \end{pmatrix}}$$

$$[R_\alpha] = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

$$[R_\beta] = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix}$$



$$\begin{pmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin\alpha+\beta & \cos(\alpha+\beta) \end{pmatrix} \begin{bmatrix} R_{\alpha+\beta} \end{bmatrix} = [R_\beta][R_\alpha]$$

$$R_{\alpha+\beta} = R_\beta \circ R_\alpha$$

$$\begin{pmatrix} C(\beta) & -S(\beta) \\ S(\beta) & C(\beta) \end{pmatrix} \begin{pmatrix} C(\alpha) & -S(\alpha) \\ S(\alpha) & C(\alpha) \end{pmatrix} = \begin{matrix} C\alpha C\beta - S(\alpha)S(\beta) & \cdots \\ C(\alpha)(\beta) + S(\alpha) + C(\beta) & \cdots \end{matrix}$$

$\Rightarrow \cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \cos\alpha\sin\beta + \cos\beta\sin\alpha$ $\square$