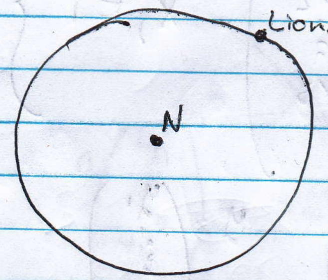
MAT 240 LECTURE 6

SEPT 24/14

READ ALONG: SEC 1.1-1.4 of TEXT.

RIDDLE ALONG:



$V_L = 4V_N$
Can N escape?

REMINDER: A "VECTOR SPACE" OVER A FIELD, F, IS A SET V (of "VECTORS") WITH AN ELEMENT 0_V AND OPERATIONS $+: V\times V \to V$ & $\times: F\times V \to V$, s.t.

VS1 $\forall x,y \in V,\ x+y = y+x$

VS2 $\forall x,y,z \in V,\ x+(y+z) = (x+y)+z$

VS3 $\forall x \in V,\ x+0_V = x$

VS4 $\forall x \in V\ \exists y \in V$ s.t. $x+y=0$

VS5 $\forall x \in V\quad 1_F \cdot x = x$

VS6 $\forall x \in V, \forall a,b \in F\quad a(bx) = (ab)x$

VS7 $a(x+y) = ax+ay$

VS8 $(a+b)x = ax+bx$

if $n=0$ list is () empty..

$F^n = \underbrace{F \times F \times F \times \cdots \times F}_{n \text{ times}}$ $= \{(x_1, x_2, \ldots, x_n) : x_i \in F\}$

$[n = 0, 1, 2, \ldots]$

EXAMPLES 1. $V = F^n$

$$= \left\{ \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_3 \end{pmatrix} : x_i \in F \right\} = \left\{ \begin{matrix} \text{column} \\ \text{vectors} \end{matrix} \right\}$$

$0_{F^n} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$ $y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$ $x+y = \begin{pmatrix} x_1+y_1 \\ x_2+y_2 \\ \vdots \\ x_n+y_n \end{pmatrix}$

[example. $\begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} -3 \\ 8 \end{pmatrix} = \begin{pmatrix} -1 \\ 13 \end{pmatrix}$
this is happening in F^2]

$cx = \begin{pmatrix} cx_1 \\ cx_2 \\ \vdots \\ cx_n \end{pmatrix}$ (example $\frac{1}{2}\begin{pmatrix} 8 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 3/2 \\ -1 \end{pmatrix}$)

Claim: This satisfies Vs1 – Vs8

Checking Vs7. $a(x+y) = a\left(\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}\right)$

$= a\begin{pmatrix} x_1+y_1 \\ \vdots \\ x_n+y_n \end{pmatrix}$ by def^{n} above

$= \begin{pmatrix} a(x_1+y_1) \\ a(x_2+y_2) \\ \vdots \\ a(x_n+y_n) \end{pmatrix}$ by def^n above

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Example 1'

$F = F_2 = \{0,1\}$ $n=8$
$\{0,1\}$ "bits"

$V = F^8 = F_2{}^8 = \{(\ )\}$
$\ni = \{(1,0,0,1,1,1,0,1)\}$ element "bytes"

"Bytes make a v.s. over bits"

$0 \cdot (1,1,1,0,1,1,0,0) = 0_V$

$$\begin{array}{r} 01001100 \\ +01111100 \\ \hline 00110000 \end{array}$$