

P32

1. Solution:

- (a) True  
let all scalars = 0.
- (b) False  
meaningless
- (c) True  
intersection = S + combinations of S.
- (d) False  
constant cannot be 0.
- (e) True
- (f) False  
eg.  $\begin{cases} a+b=1 \\ a+b=2 \end{cases}$

2. Solution:

$$(a) \Rightarrow \begin{cases} x_1 - x_2 = -1 \\ x_3 = 0 \\ x_4 = 2 \end{cases}$$

$$\Rightarrow \begin{cases} x_1 = x_2 - 1 \\ x_3 = 0 \\ x_4 = 2 \end{cases} \Rightarrow (x_1, x_2, x_3, x_4) = (x_2 - 1, x_2, 0, 2)$$

$$(b) \Rightarrow \begin{cases} x_1 = -2 \\ x_2 = -4 \\ x_3 = -3 \end{cases} \Rightarrow (x_1, x_2, x_3) = (-2, -4, -3)$$

$$c) \Rightarrow \begin{cases} x_1 + x_3 + 5x_4 = 1 \\ x_2 = x_3 - 2x_4 = 2 \\ 0 = 3 \end{cases}$$

$\Rightarrow$  no solution

$$d) \Rightarrow \begin{cases} x_1 + 8x_3 = -16 \\ x_2 - 3x_3 = 9 \\ x_4 = 2 \end{cases}$$

$$\Rightarrow \begin{cases} x_1 = -8x_3 - 16 \\ x_2 = 3x_3 + 9 \\ x_4 = 2 \end{cases}$$

$$\Rightarrow (x_1, x_2, x_3, x_4) = (-8x_3 - 16, 3x_3 + 9, x_3, 2)$$

$$e) \Rightarrow \begin{cases} x_1 - \frac{11}{3}x_4 = 6 \\ x_2 = 0 \\ x_3 - \frac{2}{3}x_4 = 1 \\ x_5 = 5 \end{cases}$$

$$\Rightarrow \begin{cases} x_1 = \frac{11}{3}x_4 + 6 \\ x_2 = 0 \\ x_3 = \frac{2}{3}x_4 + 1 \\ x_5 = 5 \end{cases}$$

$$\Rightarrow (x_1, x_2, x_3, x_4, x_5) = \left(\frac{11}{3}x_4 + 6, 0, \frac{2}{3}x_4 + 1, x_4, 5\right)$$

~~$\Rightarrow$  no solution.~~

3. Solution:

(a) Yes

$$(-2, 0, 3) = 4(1, 3, 0) - 3(2, 4, -1)$$

(b) Yes

$$(1, 2, -3) = 5(-3, 2, 1) + 8(2, -1, -1)$$

(c) No

(d) No

(e) No

(f)  $(-2, 2, 2) = 4(1, 2, -1) + 2(-3, -3, 3)$

4. Solution:

(a) Yes

$$x^3 - 3x + 5 = 3(x^3 + 2x^2 - x + 1) - 2(x^3 + 3x^2 - 1)$$

(b) No