W_3 is closed under addition.

$\because a(a_1, \frac{2a_1+a_3}{7}, a_3) = (aa_1, \frac{2aa_1+aa_3}{7}, aa_3) \in R^3$.

$\therefore (aa_1, \frac{2aa_1+aa_3}{7}, aa_3) \in W_3$ $\quad \frac{2aa_1+aa_3}{7} = \frac{2(aa_1)+(aa_3)}{7}$

W_3 is closed under addition.

$\therefore W_3$ is a subspace of R^3

d) Yes

Let $a_1=a_2=a_3=0$, $a_1-4a_2-a_3=0$.

$\Rightarrow (0,0,0) \in R^3 \Rightarrow (0,0,0) \in W_4$

$\Rightarrow 0 \in W_4$

$a_3 = a_1 - 4a_2$

Assume a_1', a_2' s.t. $(a_1', a_2', a_1'-4a_2') \in R^3$

$\Rightarrow (a_1', a_2', a_1'-4a_2') \in W_4$

$\because (a_1, a_2, a_1-4a_2) + (a_1', a_2', a_1'-4a_2')$

$= (a_1+a_1', a_2+a_2', a_1+a_1'-4a_2-4a_2') \in R^3$, $a_1+a_1'-4a_2-4a_2'$

$\therefore (a_1+a_1', a_2+a_2', a_1+a_1'-4a_2-4a_2') \in W_4$. $=(a_1+a_1')-4(a_2+a_2')$

W_4 is closed under addition.

$\because a(a_1, a_2, a_1-4a_2) = (aa_1, aa_2, aa_1-4aa_2) \in R^3$

$aa_1-4aa_2 = (aa_1)-4(aa_2)$ $\quad \therefore (aa_1, aa_2, aa_1-4aa_2) \in W_4$.

W_4 is closed under multiplication.

$\therefore W_4$ is a subspace of R^3!

e) No.

$\because$ let $a_1 = a_2 = 0 \Rightarrow a_3 = -\frac{1}{3} \neq 0$

$\therefore (0,0,0) \notin W_5 \quad \Rightarrow 0 \notin W_5$

$\therefore W_5$ is not a subspace of R^3

f) No.

let $a_1 = a_2 = a_3 = 0$, $(0,0,0) \in R^3 \Rightarrow (0,0,0) \in W_6$

$\Rightarrow 0 \in W_6$

$5a_1^2 - 3a_2^2 + 6a_3^2 = 0$, then we have 2 elements:

$(a_1, a_2, a_3), (a_1', a_2', a_3') \in W_6$

$\therefore (a_1, a_2, a_3) + (a_1', a_2', a_3')$

$= (a_1 + a_1', a_2 + a_2', a_3 + a_3')$

$5(a_1 + a_1')^2 - 3(a_2 + a_2')^2 + 6(a_3 + a_3')^2$

$= (5a_1^2 - 3a_2^2 + 6a_3^2) + (5a_1'^2 - 3a_2'^2 + 6a_3'^2) + 10a_1a_1' - 6a_2a_2' + 12a_3a_3'$

$= 0 + 0 + 10a_1a_1' - 6a_2a_2' + 12a_3a_3' = 10a_1a_1' - 6a_2a_2' + 12a_3a_3' \neq 0$

$\therefore (a_1 + a_1', a_2 + a_2', a_3 + a_3') \notin W_6$

$\therefore W_6$ is not closed under addition.

$a(a_1, a_2, a_3) = (aa_1, aa_2, aa_3) \in R^3$

$5(aa_1)^2 - 3(aa_2)^2 + 6(aa_3)^2 = a^2(5a_1^2 - 3a_2^2 + 6a_3^2) = a^2 \cdot 0 = 0.$

W_6 is closed under multiplication.

Hence W_6 isn't a subspace of R^3