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By r/c interchanges, can bring the $\neq 0$ entry

$$\begin{pmatrix} \neq 0 & ? \\ ? & ? \\ \vdots & \vdots \end{pmatrix}$$

By multiplying first row by something
can get to

$$\begin{pmatrix} 1 & ? \\ ? & ? \\ \vdots & \vdots \end{pmatrix}$$

By col ops of type 3, get to

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ ? & ? & & \end{pmatrix}$$

By row ops of type 3, get to

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & & \\ 0 & & & \end{pmatrix}$$

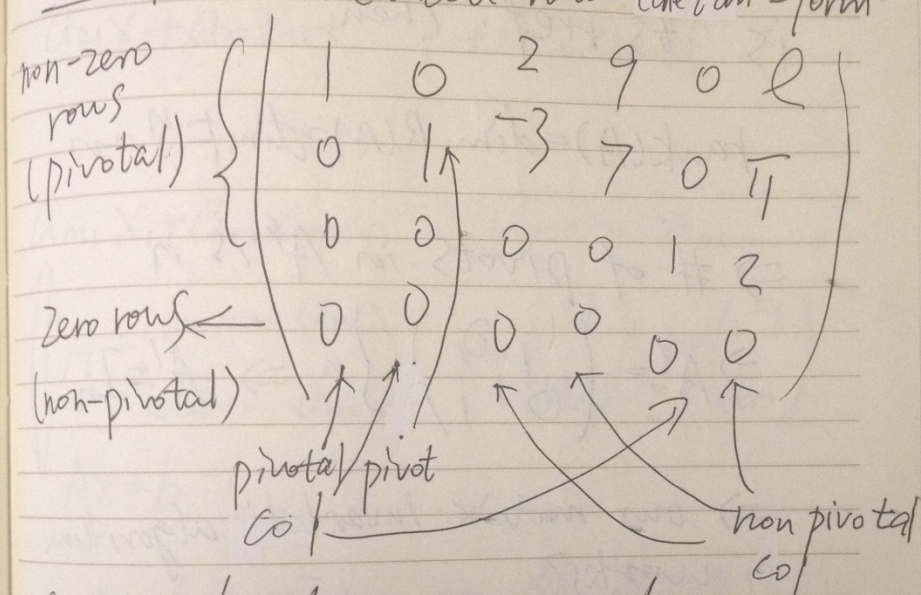
do the same to A'

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & A'' & \end{array} \right)$$

Using only row ops

- R
R
E
F
- * the first non-zero entry in every row is a 1. (the "pivot")
 - * In the column of a pivot, everything else is 0.
 - * Going down the rows, the pivots are further & further to the right
 - * all 0 rows are at bottom

Example "reduced-row-echelon form"



Now w/ col ops can reach

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \rightsquigarrow \left(\begin{array}{c|c} I_k & 0 \\ \hline 0 & 0 \end{array} \right) \quad m$$

ts
ght

conclusions:

1. If A is rref, rank(A) is the number of pivots in A