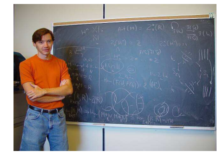
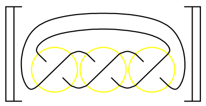
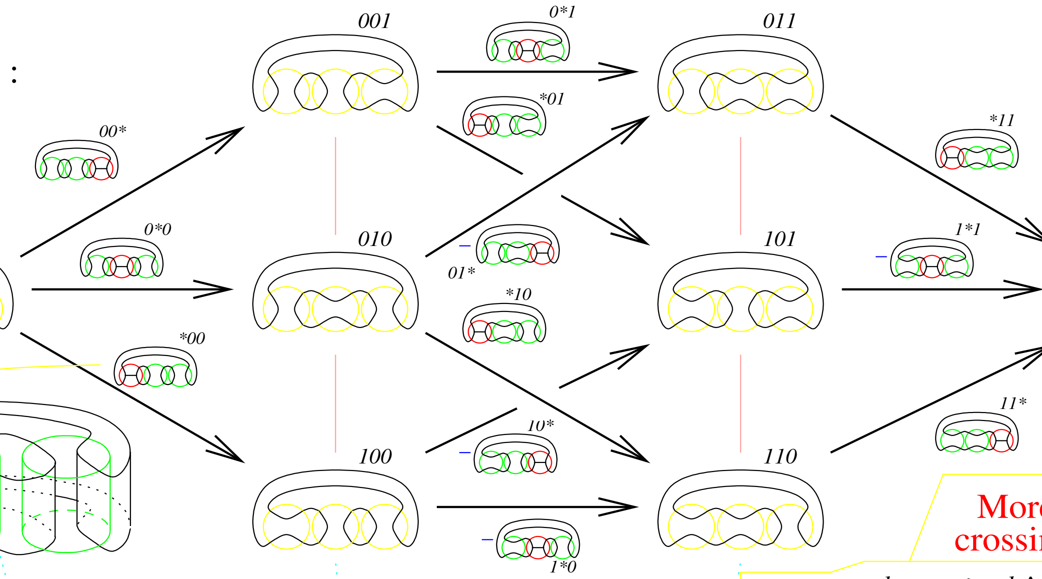
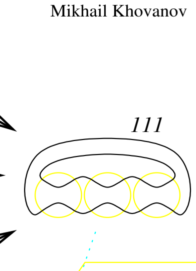
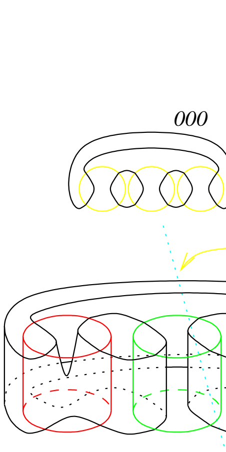
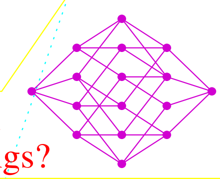
Khovanov Homology

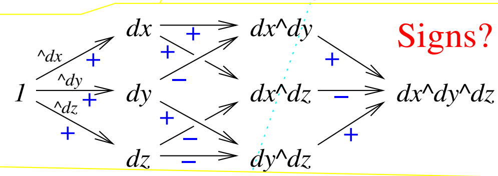






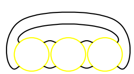
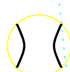
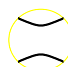
More crossings?

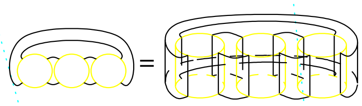
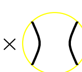
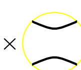
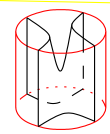


Signs?

What is it?

A cube for each knot/link projection;

Vertices: All fillings of  with  or with  .

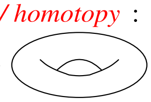
Edges: All fillings of $I\times$  with $I\times$  or with $I\times$  and precisely one  .

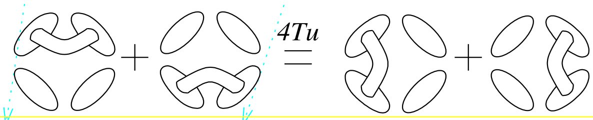
Where does it live?

In *Kom(Mat(<Cob> / {S, T, 4Tu})) / homotopy* :

Kom: Complexes *Cob: Cobordisms*

<...>: Formal lin. comb. *Mat: Matrices*

S:  *=0* *T:*  *=2*



Jones/Kauffman?

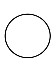
$$V^{\otimes 3} \longrightarrow (V^{\otimes 2}\oplus V^{\otimes 2}\oplus V^{\otimes 2})\{1\} \longrightarrow (V\oplus V\oplus V)\{2\} \longrightarrow V^{\otimes 2}\{3\}$$

$$(q+q^{-1})^3 \qquad 3q(q+q^{-1})^2 \qquad 3q^2(q+q^{-1}) \qquad q^3(q+q^{-1})^2$$

V. Jones

L. Kauffman

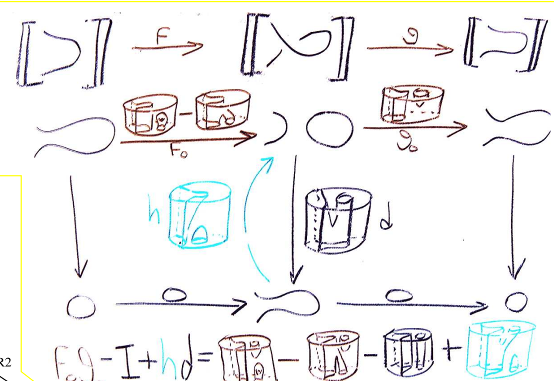
A TQFT takes it to a complex whose graded Euler characteristic is the Jones polynomial.

The key point:  $\longrightarrow$ $V = \langle v_+, v_-\rangle$, $\deg v_\pm = \pm 1$

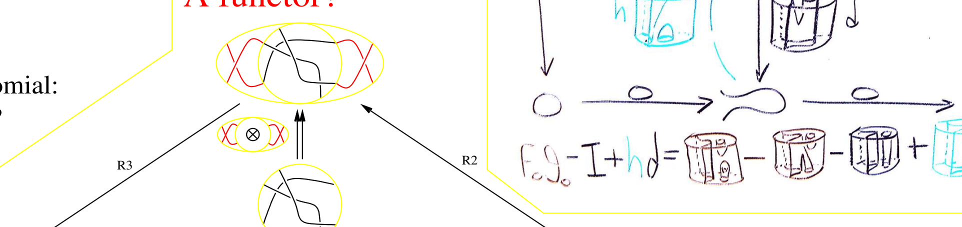
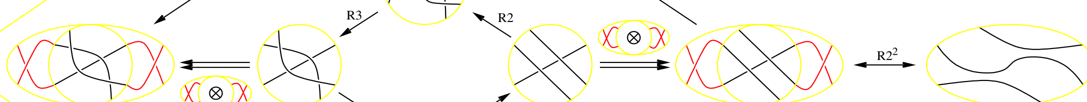
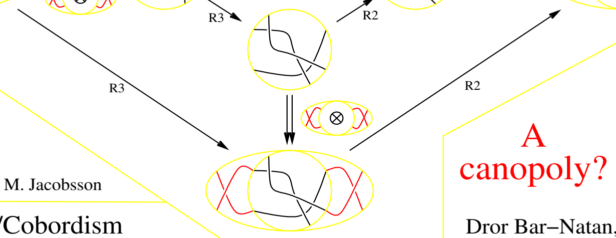
$q\text{-dim}V = q+q^{-1}$

But is it invariant?

(With similar proofs for R–II and R–III)



A functor?

Why is it interesting?

1. It is stronger than the Jones polynomial.
2. It is less understood than the Jones polynomial:
   a. Does it have a topological interpretation?
   b. Does it have a "physical" interpretation?
   c. Does it also work for other quantum invariants?
   d. Does it work for manifolds and for knots in manifolds?
   e. Is there a relation with finite–type invariants?
   f. Does it work for "virtual knots"?
3. Jacobsson, Khovanov: It is a functor!!! (from knots and cobordisms to complexes and morphisms)

M. Jacobsson

See http://www.math.toronto.edu/~drorbn/papers/Cobordism

A canopoly?

Dror Bar–Natan, Warszawa, July 2003.

More at http://www.math.toronto.edu/~drorbn/Talks/UWO-040213/