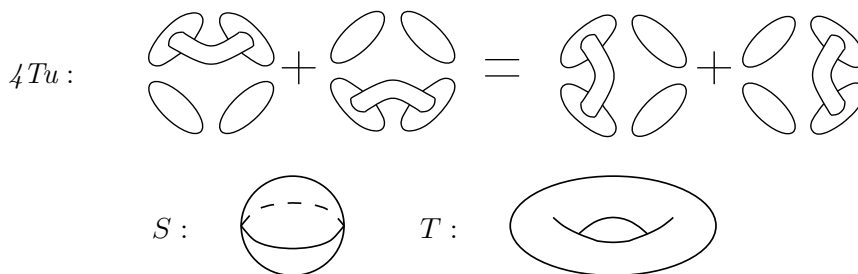


DROR BAR-NATAN



**What is it good for?**

(1) Cutting necks:



(2) Recovers the good old Khovanov theory,

$$\begin{aligned} \mathcal{F}(\bowtie) &= \epsilon : \begin{cases} 1 \mapsto v_+ \end{cases} & \mathcal{F}(\ominus) &= \eta : \begin{cases} v_+ \mapsto 0 \\ v_- \mapsto 1 \end{cases} \\ \mathcal{F}(\bowtie) &= \Delta : \begin{cases} v_+ \mapsto v_+ \otimes v_- + v_- \otimes v_+ \\ v_- \mapsto v_- \otimes v_- \end{cases} & \mathcal{F}(\circ\circ) &= m : \begin{cases} v_+ \otimes v_- \mapsto v_- & v_+ \otimes v_+ \mapsto v_+ \\ v_- \otimes v_+ \mapsto v_- & v_- \otimes v_- \mapsto 0. \end{cases} \end{aligned}$$

- (3) Trivially extends to tangles.
- (4) Well suited to prove invariance for cobordisms.
- (5) Recovers Lee’s theory,

$$\Delta : \begin{cases} v_+ \mapsto v_+ \otimes v_- + v_- \otimes v_+ \\ v_- \mapsto v_- \otimes v_- + v_+ \otimes v_+ \end{cases} \quad m : \begin{cases} v_+ \otimes v_- \mapsto v_- & v_+ \otimes v_+ \mapsto v_+ \\ v_- \otimes v_+ \mapsto v_- & v_- \otimes v_- \mapsto v_+. \end{cases}$$

(6) Leads to a new theory (over  $\mathbb{Z}/2$  and with  $\deg h = -2$ ),

$$\Delta : \begin{cases} v_+ \mapsto v_+ \otimes v_- + v_- \otimes v_+ + hv_+ \otimes v_+ \\ v_- \mapsto v_- \otimes v_- \end{cases} \quad m : \begin{cases} v_+ \otimes v_- \mapsto v_- & v_+ \otimes v_+ \mapsto v_+ \\ v_- \otimes v_+ \mapsto v_- & v_- \otimes v_- \mapsto hv_- \end{cases}$$

- (7) Trivially extends to knots on surfaces.
- (8) Non-trivially recovers Khovanov’s  $c$ ,

$$\begin{aligned} \epsilon : \begin{cases} 1 \mapsto v_+ \end{cases} & \eta : \begin{cases} v_+ \mapsto 0 \\ v_- \mapsto -c \end{cases} \\ \Delta : \begin{cases} v_+ \mapsto v_+ \otimes v_- + v_- \otimes v_+ + cv_- \otimes v_- \\ v_- \mapsto v_- \otimes v_- \end{cases} & m : \begin{cases} v_+ \otimes v_- \mapsto v_- & v_+ \otimes v_+ \mapsto v_+ \\ v_- \otimes v_+ \mapsto v_- & v_- \otimes v_- \mapsto 0. \end{cases} \end{aligned}$$

(Added June 29, 2004: what appeared to work didn’t quite. The recovery of Khovanov’s  $c$  remains open).

“God created the knots, all else in topology is the work of man.”

Leopold Kronecker (modified)

URL: <http://www.math.toronto.edu/~drorbn/papers/Cobordism> (and see the ‘‘GWU’’ handout)

Date: May 30, 2004.