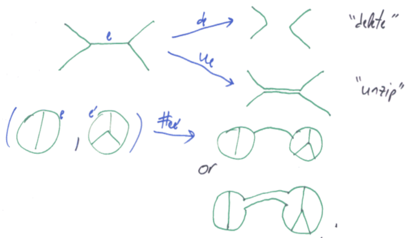
So many interesting properties of knots are definable using Knotted Trivalent Graphs (KTGs) (Fully labeled, oriented, Framed:)

and the basic operations between them:



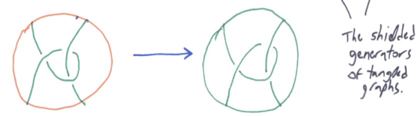
We seek a "TG-morphism" into algebra:

1. $\forall\Gamma$ an algebraic space $A(\Gamma)$, $Z_\Gamma: K(\Gamma)\to A(\Gamma)$.
2. $d, u, \#$ defined on the $A(\Gamma)$'s.
3. $K(\Gamma) \xrightarrow{Z} A(\Gamma)$, $\downarrow u_e$, $K(u_e\Gamma) \xrightarrow{Z} A(u_e\Gamma)$ etc.

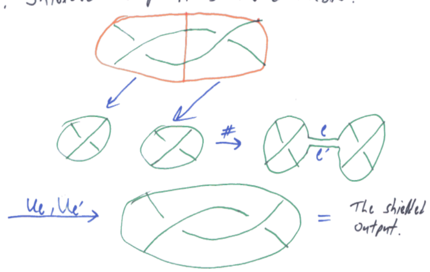
Aside 1

KTG is finitely generated, by (tetrahedron) and (square):

1. "Shield" all tangles:



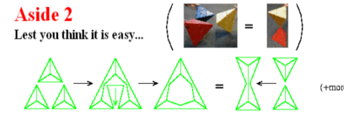
2. Shielded compositions are definable:



3. Relations: 1. whatever makes this well-defined. 2. The Reidemeister moves.
4. So finding a Z is just a matter of finding/guessing $Z(\triangle)$ & $Z(\boxtimes)$, solving a few relations....

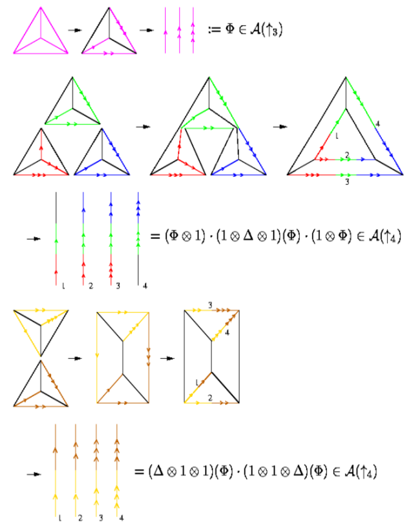
Aside 2

Lest you think it is easy...



Claim. With $\Phi := Z(\triangle)$, the above relation becomes equivalent to the Drinfel'd's pentagon of the theory of quasi Hopf algebras.

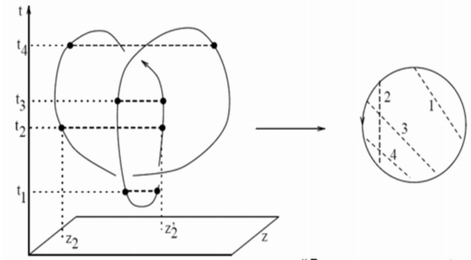
Proof.



Abstract [edit]

We construct a (very) well-behaved invariant of knotted trivalent graphs using only the Kontsevich integral, in three steps.

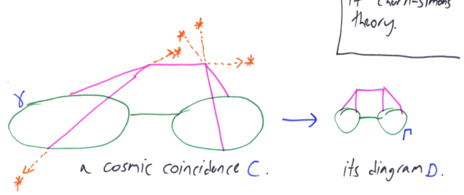
Step 1 - The Naive Kontsevich Integral [edit]



$$Z_0(K) = \sum_{m;\ t_1<\ldots<t_m;\ P=\{(z_i,z_i')\}} \frac{(-1)^{\#P_\downarrow}}{(2\pi i)^m} D_P \bigwedge_{i=1}^m \frac{dz_i - dz_i'}{z_i - z_i'}$$

We define the "naive Kontsevich integral" Z_1 of a knotted trivalent graph or a slice thereof as in the "standard" picture above, except generalized to graphs in the obvious manner.

Cosmic Coincidences:



Definition (Dylan)

$Z(\gamma) = \sum_C D \in \mathcal{A}(\Gamma) :=$ ⟨ ⟩ / AS: ... IHX: ... STU: ... Vinv: ... = 0

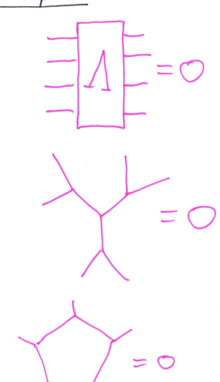
(lots of work is hidden here, and some unknowns)

Behavior under / → // is predictable.

Internal Quotients

Involve only chords and no strands.

Examples:



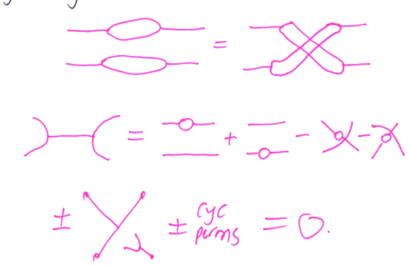
1. "The Lie algebra is 3-dimensional" (i.e., it is $sl(2)$...)
2. "Diagrams are (very) shallow" (related to $gl(1|1)$, Alexander)
3. "Pentagons stay out"

4, 5, ... Use your imagination.

Classification?

Theorem There is a minimal quotient containing the Alexander polynomial.

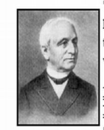
Proof Use the "internal kernel" of the Alexander weight system:



What remains is a polynomial amount of information!

Conjecture This explains everything that we know about the Alexander polynomial.

Some propaganda...



"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

Visit!

Edit!

I don't understand... (more)

- Who needs "trivalent" in "knotted trivalent graphs"? (more)
- The notion of "framing". (more)
- Configuration space integrals / perturbative Chern-Simons theory for knotted graphs. (more)
- **The Alexander polynomial.** (more)
- The Lieberum "$gl(1|1)$" associator. (more)
- Must associators be so hard? Why? (more)
- The relationship between "genus" and "finite type". (more)
- The relationship between "gropes" and "finite type". (more)
- ***TG*-ideals / internal quotients.**
- What exactly are *TG*-algebras? What are the syzygies among the relations between their generators?
- Virtual knots.
- Quantum groups(!) (more)
- The work of Etingof and Kazhdan.
- The polynomiality of knot polynomials.
- Functional equations.
- The Eilenberg-Zilber theorem.
- Which other interesting classes of knots/links are "TG-definable"?

More at http://www.math.toronto.edu/~drorbn/Talks/Copenhagen-081009/