Cosmic Coincidences and Several Other Stories, 1

Dror Bar−Natan at the University of Tennessee
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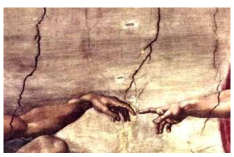
Abstract. In the first half of my talk I will tell a cute and simple story — how given a knot in $\mathbb{R}^3$ one may count all possible "cosmic coincidences" associated with that knot, and how this count, appropriately packaged, becomes an invariant Z with values in some space $\mathcal{A}$ of linear combinations of certain trivalent graphs.
In the second half of my talk I will describe (rather sketchily, I'm afraid) a part of the story surrounding Z and $\mathcal{A}$: How the same Z also comes from quantum field theory, Feynman diagrams, and configuration space integrals. How $\mathcal{A}$ is a space of universal formulas which make sense in every metrized Lie algebra and how specific choices for that Lie algebra correspond to various famed knot invariants. How Z solves a universal topological problem, and how solving for Z is solving some universal Lie-algebraic problem. All together, this is the u-story.
In the remaining time I will mention several other Z's and $\mathcal{A}$'s and the parallel (yet sometimes interwoven) stories surrounding them — the v-story, and w-story, and perhaps also the p-story. Each of these stories is clearly still missing some chapters.

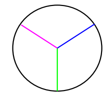
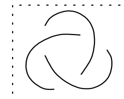
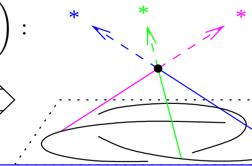
Creation of Adam



Michelangelo

Disclaimer
We'll concentrate on the beauty and ignore the cracks.

$\langle D, K\rangle_{\text{Stonehenge}} :=$ (The signed Stonehenge pairing of D and K) :

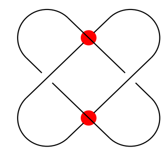
$D=$  $K=$  $\Rightarrow$ 

count with signs

The Gaussian linking number

$lk(\text{link}) = \sum_{\text{vertical chopsticks}} (\text{signs})$

$= \langle \text{O—O}, \text{link} \rangle_{\text{Stonehenge}}$



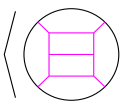
C.F. Gauss

The generating function of all cosmic coincidences:

D. Thurston

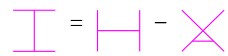
$$Z(K) := \lim_{N\to\infty} \sum_{3\text{-valent } D} \frac{\langle D, K\rangle_{\text{Stonehenge}} D}{2^c c! \binom{N}{e}} \cdot \left(\begin{array}{c}\text{framing-}\\\text{dependent}\\\text{counter-term}\end{array}\right) \in \mathcal{A}(\circlearrowleft)$$

N := # of stars
c := # of chopsticks
e := # of edges of D

$\mathcal{A}(\circlearrowleft)$:=Span $\left\langle\right.$  $\left.\right\rangle$ / oriented vertices AS: $\curlyvee + \curlyvee = 0$ & more relations

When deforming, catastrophes occur when:

A plane moves over an intersection point − Solution: Impose IHX,



(see below)

An intersection line cuts through the knot − Solution: Impose STU,

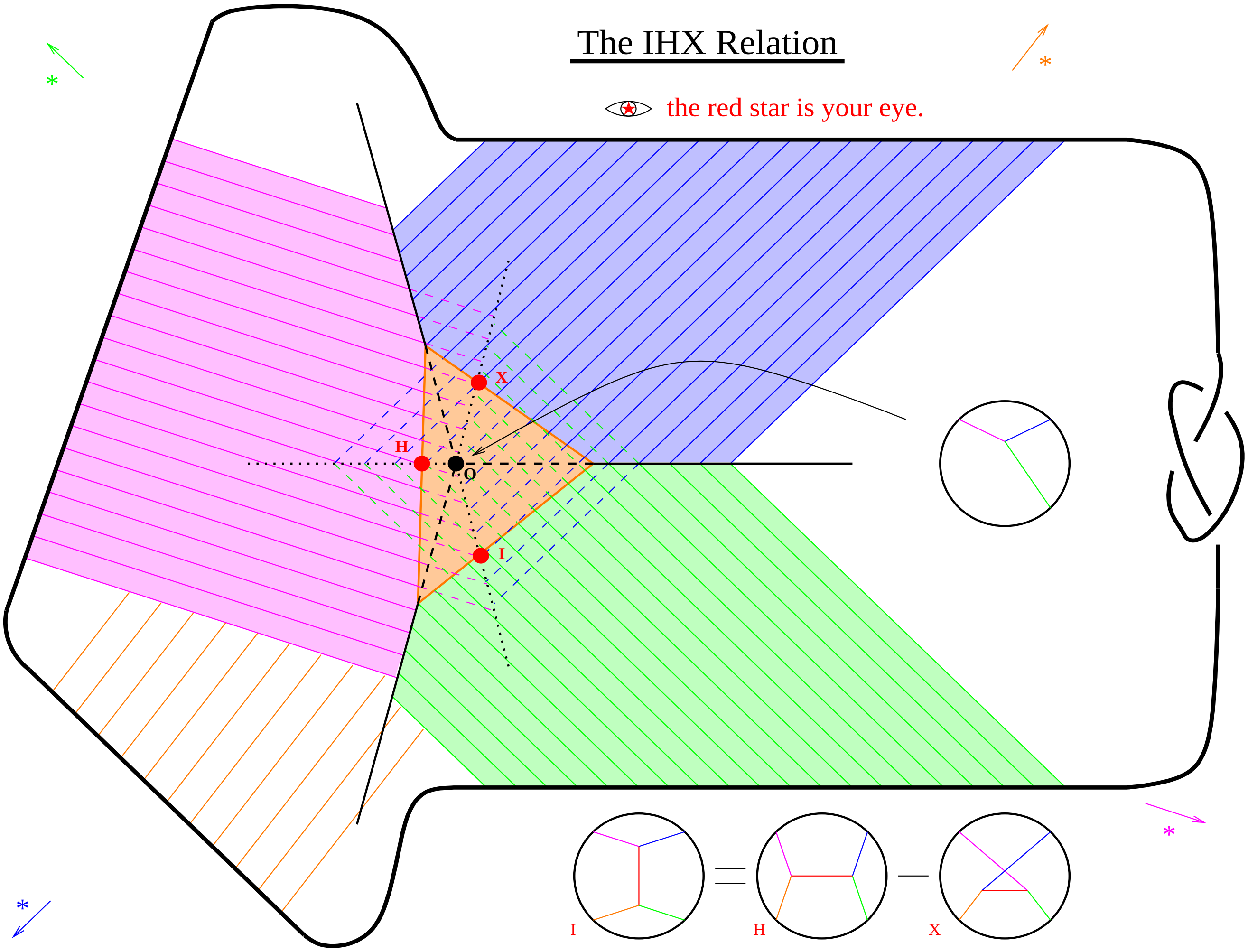
(similar argument)

The Gauss curve slides over a star − Solution: Multiply by a framing−dependent counter−term.

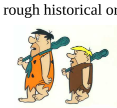
(not shown here)

Theorem. Modulo Relations, $Z(K)$ is a knot invariant!

The IHX Relation



The Cast
in rough historical order



The Neolithic People

Carl Friedrich Gauss
Edward Witten
Victor Vassiliev
Mikhail Goussarov

Maxim Kontsevich

Raoul Bott

Clifford Taubes

Thang Le

Jun Murakami

Tomotada Ohtsuki

Video and more at http://www.math.toronto.edu/~drorbn/Talks/Tennessee-1103/ and at http://www.math.toronto.edu/~drorbn/Talks/Caen-1206/#Colloquium