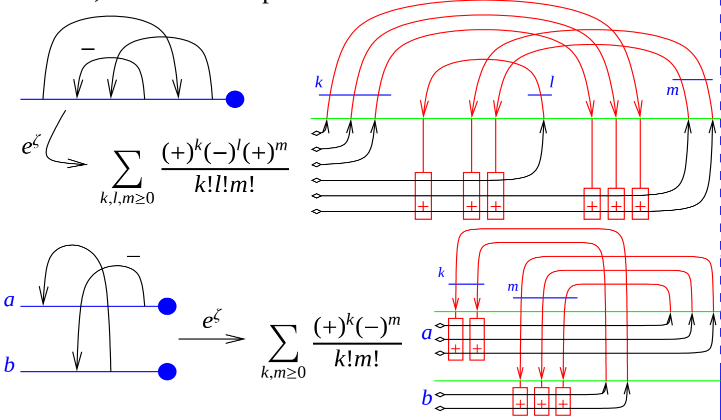
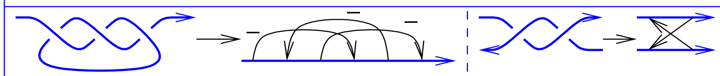
A Partial Reduction of BF Theory to Combinatorics, 2

Theorem 1 (with Cattaneo, Dalvit (credit, no blame)). In the ribbon case, e^ζ can be computed as follows:





Theorem 2. Using Gauss diagrams to represent knots and T-component pure tangles, the above formulas define an invariant in $CW(FL(T)) \to CW(T)$, "cyclic words in T".
• Agrees with BN-Dancso [BND] and with [BN2]. • In-practice computable! • Vanishes on braids. • Extends to w. • Contains Alexander. • The "missing factor" in Levine's factorization [Le] (the rest of [Le] also fits, hence contains the MVA). • Related to / extends Farber's [Fa]? • Should be summed and categorified.

References.

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Continuing Joost Slingerland...

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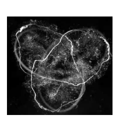
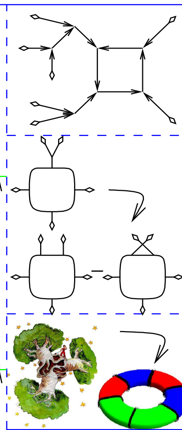
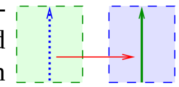
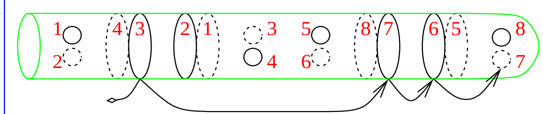




Sketch of Proof. In $4D$ axial gauge, only "drop down" red propagators, hence in the ribbon case, no M-trivalent vertices. S integrals are ± 1 iff "ground pieces" run on nested curves as below, and exponentials arise when several propagators compete for the same double curve. And then the combinatorics is obvious...

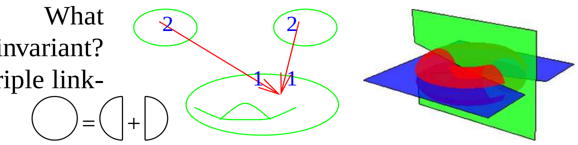




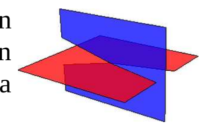
Musings

Chern-Simons. When the domain of BF is restricted to ribbon knots, and the target of Chern-Simons is restricted to trees and wheels, they agree. Why?

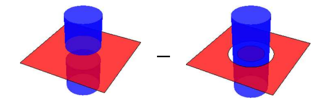
Is this all? What about the ∨-invariant? (the "true" triple linking number)



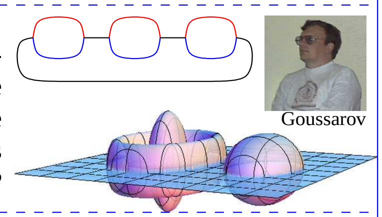
Gnots. In 3D, a generic immersion of S^1 is an embedding, a knot. In 4D, a generic immersion of a surface has finitely-many double points (a gnot?). Perhaps we should be studying these?



Finite type. What are finite-type invariants for 2-knots? What would be "chord diagrams"?

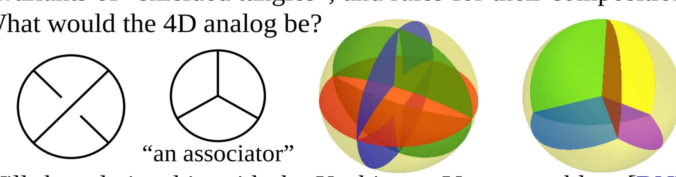


Bubble-wrap-finite-type. There's an alternative definition of finite type in 3D, due to Goussarov (see [BN1]). The obvious parallel in 4D involves "bubble wraps". Is it any good?



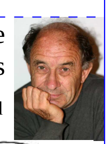
Goussarov

Shielded tangles. In 3D, one can't zoom in and compute "the Chern-Simons invariant of a tangle". Yet there are well-defined invariants of "shielded tangles", and rules for their compositions. What would the 4D analog be?



Will the relationship with the Kashiwara-Vergne problem [BND] necessarily arise here?

Plane curves. Shouldn't we understand integral / finite type invariants of plane curves, in the style of Arnold's J^+, J^-, and St [Ar], a bit better?



Arnold

	a(⨯)	a(crossing)	a(crossing)	∞	○	1 curl	2 curls	3 curls	···
St	1	0	0	0	0	1	2	3	···
J^+	0	2	0	0	0	−2	−4	−6	···
J^-	0	0	−2	−1	0	−3	−6	−9	···

"God created the knots, all else in topology is the work of mortals."
Leopold Kronecker (modified)

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Video and more at http://www.math.toronto.edu/~drorbn/Talks/Vienna-1402/